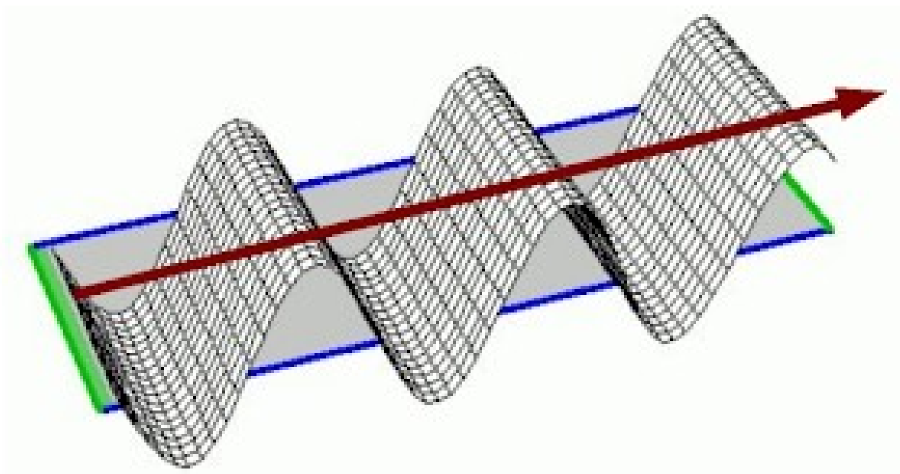
Swarm measurements and lightning activity: minimum variance and inter-satellite cross-correlation analysis

CBK, OBSEE & AGH

Minimum variance analysis



Plane wave in the magnetic field

$$\mathbf{k} = (k_x, 0, 0) \quad \frac{\partial}{\partial y} = \frac{\partial}{\partial z} = 0$$

$$\nabla \cdot \mathbf{B} = 0 \quad \Rightarrow \quad \frac{\partial B_x}{\partial x} = 0$$

$$-\nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t} \quad \Rightarrow \quad \frac{\partial B_x}{\partial t} = 0$$

These idealized conditions can be translated to

$$\sigma^2_{B_x} \ll \max(\sigma^2_{B_y}, \sigma^2_{B_z})$$

for time series obtained from satellite measurements

Minimum variance analysis

The problem is that the frame for our dataset is usually not co-aligned with the wave frame i.e.

$$\mathbf{k} = (k_x, k_y, k_z)$$

But for a time series of measurements of the magnetic field components we can construct the covariance matrix

$$\mathrm{M}_{i,j} = \langle B_i B_j \rangle - \langle B_i \rangle \langle B_j \rangle$$

which is symmetric.

Thus solving eigenproblem for the covariance matrix

$$\mathrm{M}_{i,j} v_j = \lambda v_i$$

we get real eigenvalues and othogonal eigenvectors.

Minimum, intermediate and maximum variance directions can be identified.

Eigenvalues correspond to variances along those directions, i.e.

$$\lambda = \sigma^2$$

What do we get from minimum variance analysis?

The smallest-eigenvalue eigenvector can be interpreted as minimum variance direction. **It gives an estimation of propagation direction for plane waves** (although not the vector direction), which may give us information about wave origin. It is however problematic to interpret if we deal with propagation in nonhomogeneous medium.

Indication whether the plane wave model applies to our dataset

$$\lambda_1 \ll \max(\lambda_2, \lambda_3)$$

From minimum variance vectors we can construct a new orthonormal vector base. If we rotate our time series to the minimum variance frame (i.e. wave frame), we can plot a hodograph from B_2,B_3 components and study wave polarization.

This analysis method is universal, it can be applied to both harmonic waves and isolated structures, although we need to be careful with choosing properly time span for the analysis.

Interpretation of results

By solving the eigenproblem for the covariance matrix

$$\mathrm{M}_{i,j} v_j = \lambda v_i \qquad \lambda = \sigma^2$$

We can get the following generic cases:

$\lambda_1 \ll \lambda_2 \ll \lambda_3$ Well separated eigenvalues, e.g. elliptically polarized wave

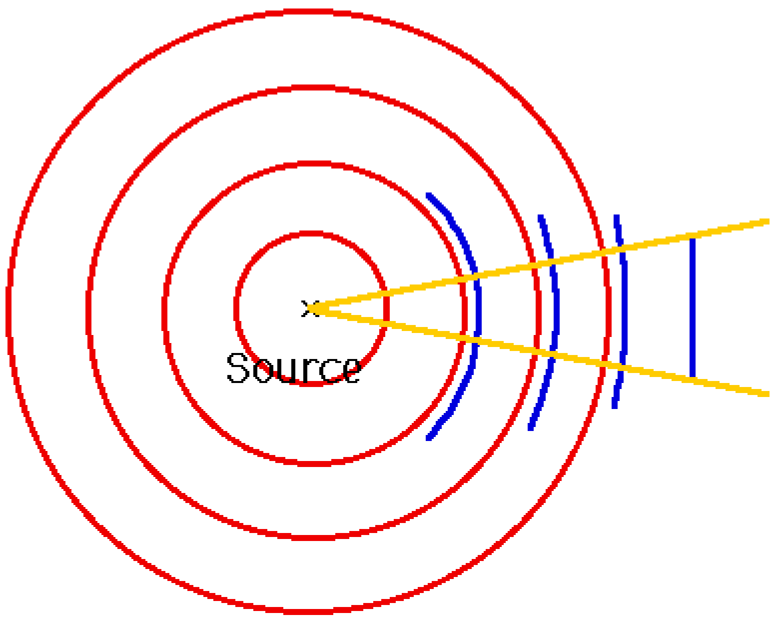
$\lambda_1 \ll \lambda_2 \approx \lambda_3$ Well separated smallest eigenvalue, e.g. circularly polarized wave

$\lambda_1 \approx \lambda_2 \ll \lambda_3$ Well separated largest eigenvalue, linearly polarized wave

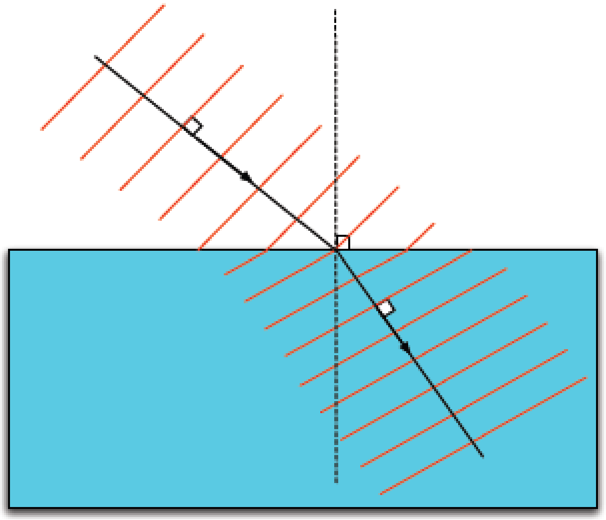
$\lambda_1 \approx \lambda_2 \approx \lambda_3$ No minimum variance direction, plane-wave model does not apply

Comments on the wave-propagation direction

Spherical wave front can be locally approximated as plane wave



Non-homogeneous background for wave propagation (e.g. density or magnetic field gradients) can change the normal direction of a wave front



Comments on the wave polarization

Linear, ecliptic or circular polarization may indicate what kind of wave we are dealing with

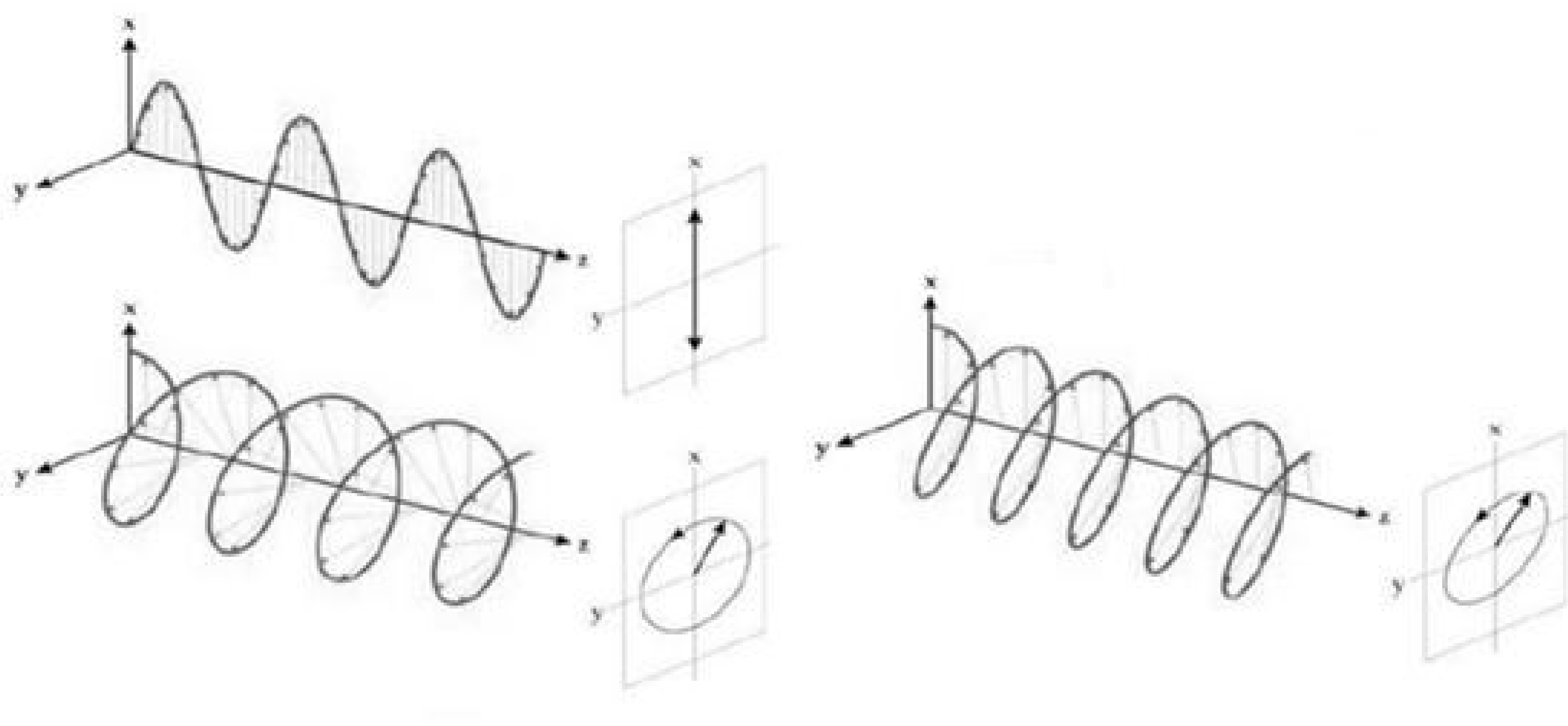
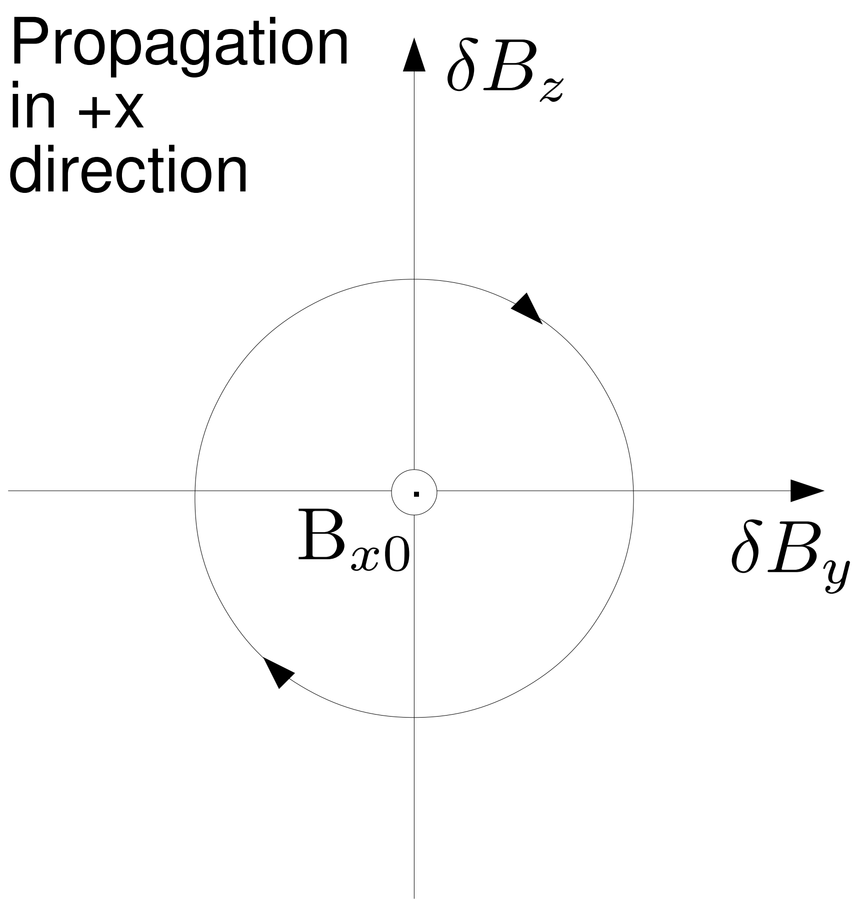
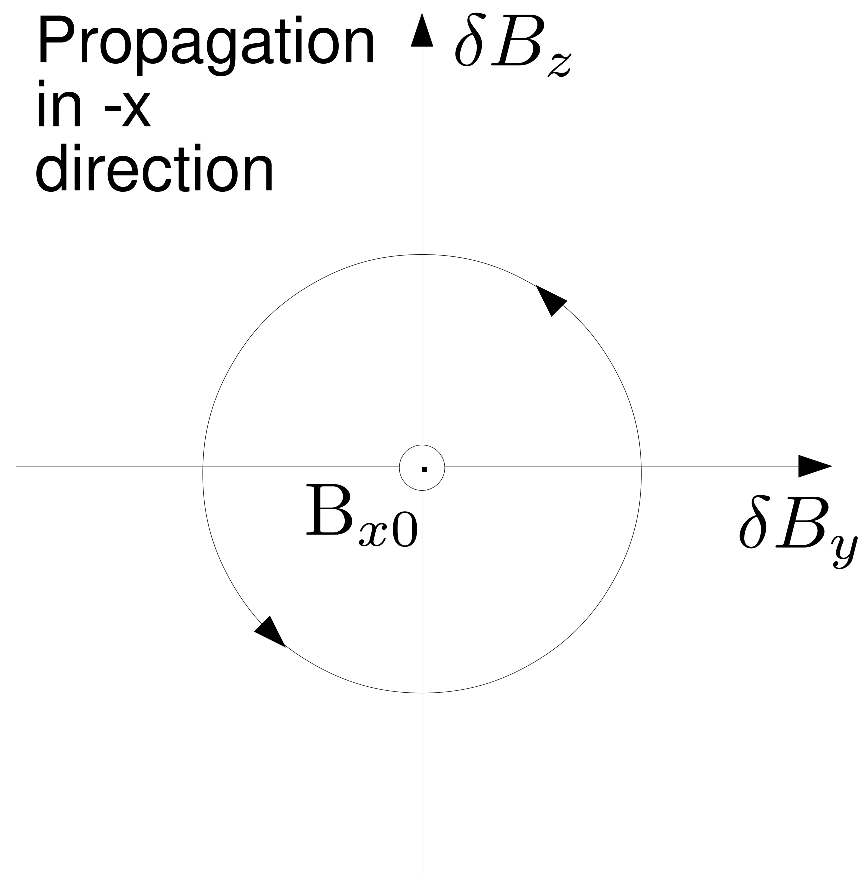


Figure 8 : Graphical illustration of linearly, circularly and elliptically polarized EM waves.

Since the minimum variance analysis itself does not give any information on the vector direction, thus polarization may be helpful in finding where the wave comes from

Polarization of Alfven wave





If the angle between minimum variance axis and magnetic field is <90°, we can infer the vector direction of the wave propagation (provided we are dealing with Alfven wave)

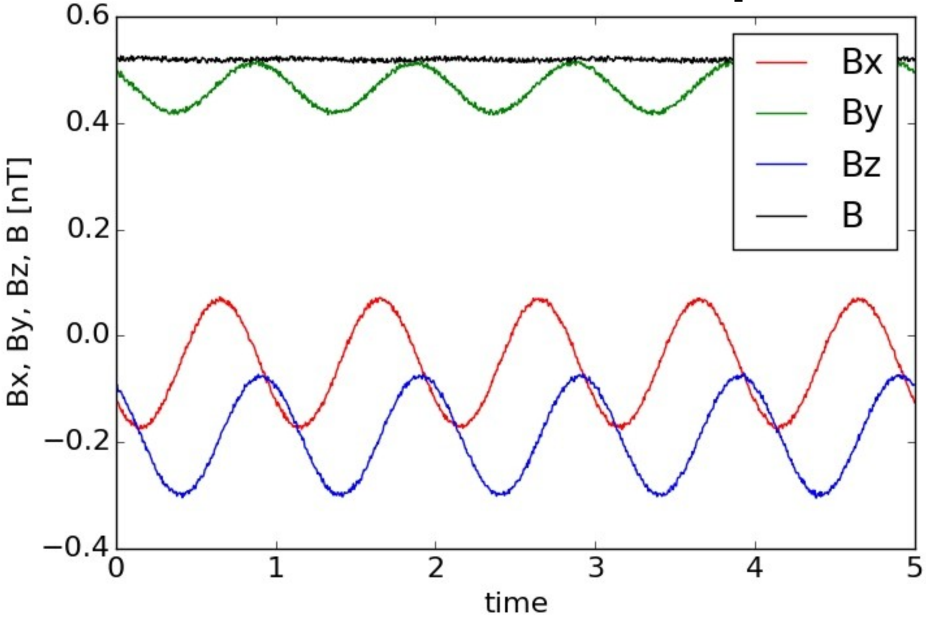
Uncertainties estimation

Bootstrap method (Kawano and Higuchi, 1995): large number of minimum variance calculations with bootstrap data samples

Bootstrap samples generated from n vectors of the measured magnetic field by drawing n elements with replacement

N=1000 bootstrap samples used to find standard deviations of the components of the minimum variance frame, angle between minimum variance vector and magnetic field, etc.

Testing on synthetic dataset – circular polarization

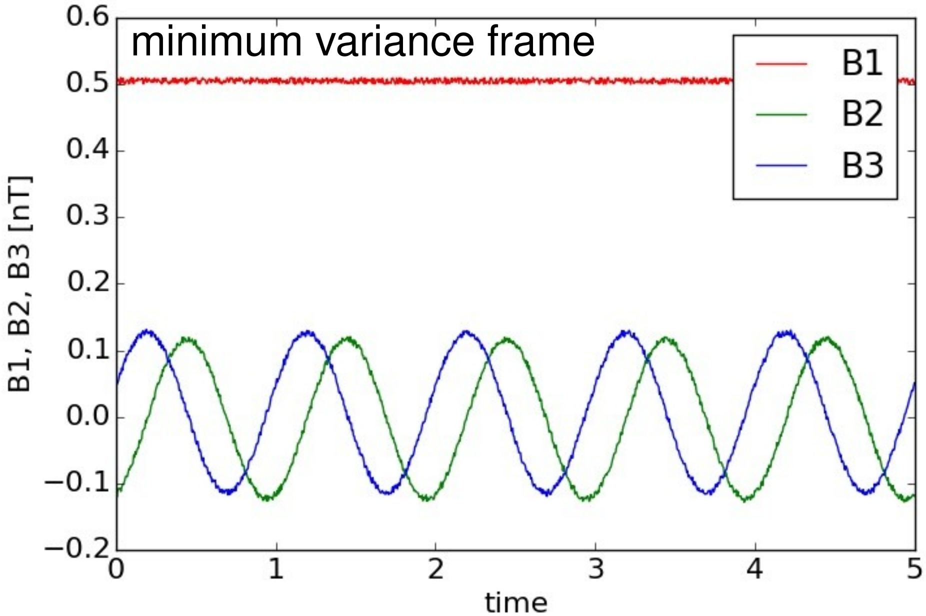


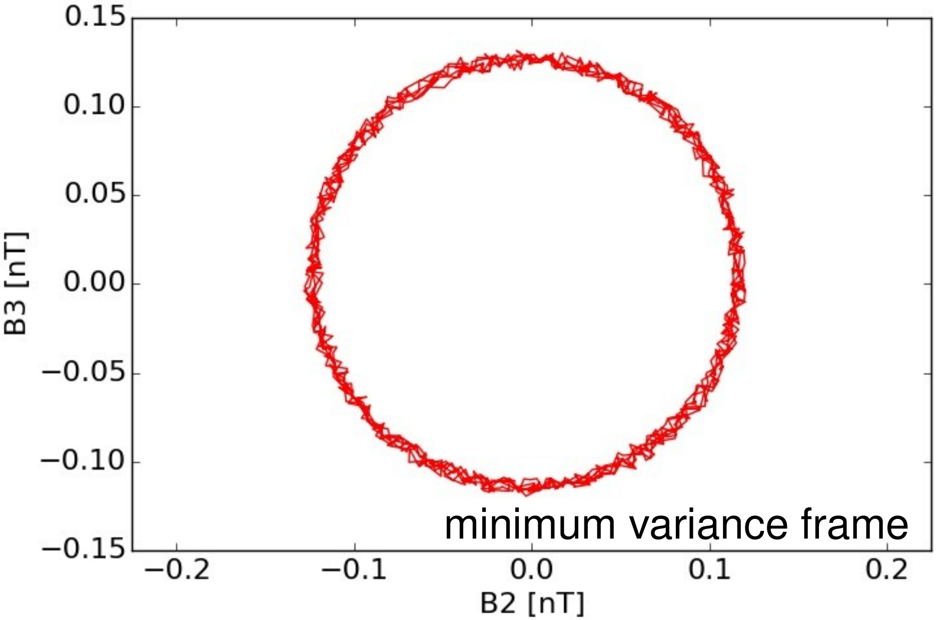
Generate time series, add noise then transform it to a randomly oriented frame

$$B_x = B_0$$

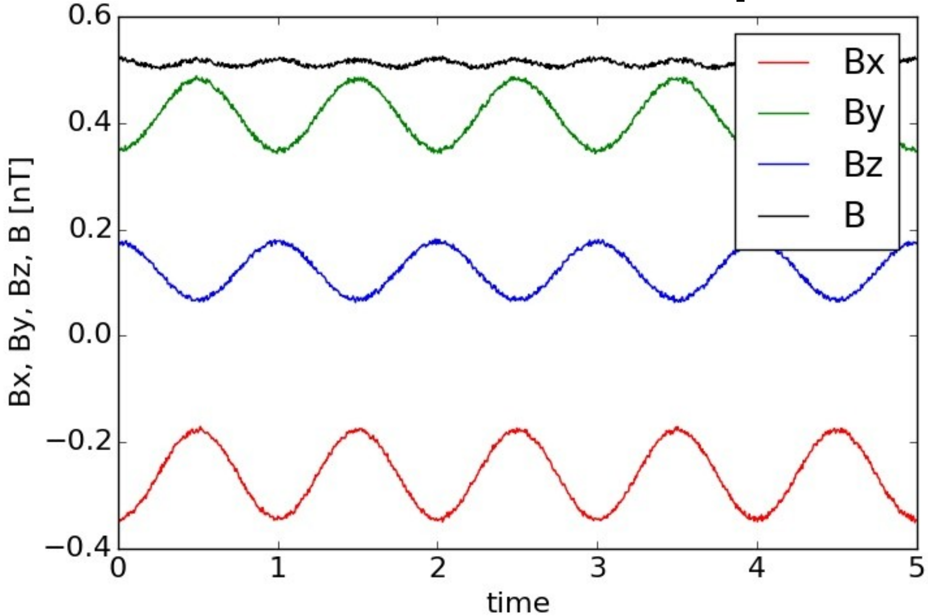
$$B_y = \delta B \sin(2\pi t)$$

$$B_z = \delta B \cos(2\pi t)$$





Testing on synthetic dataset – linear polarization

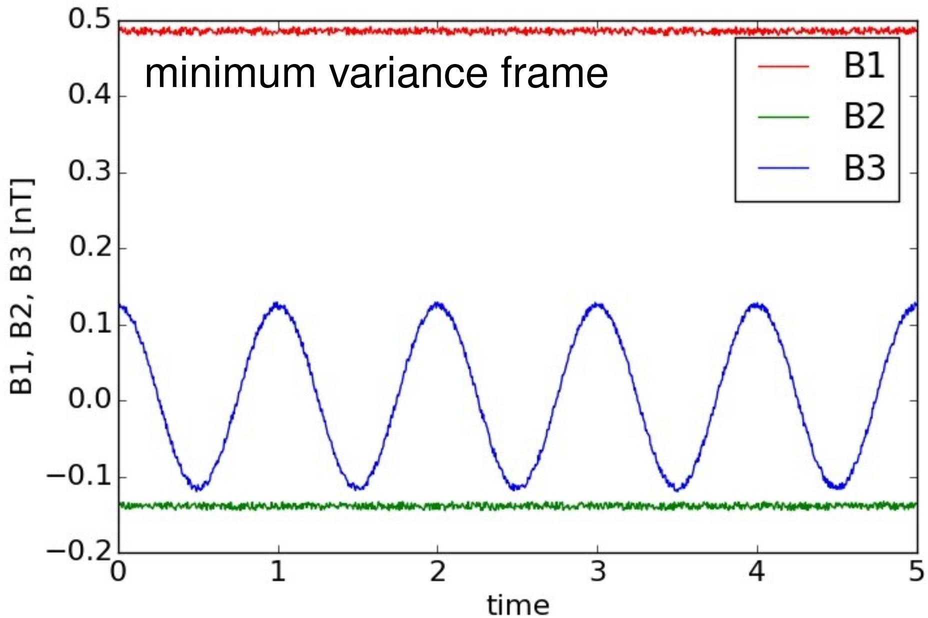


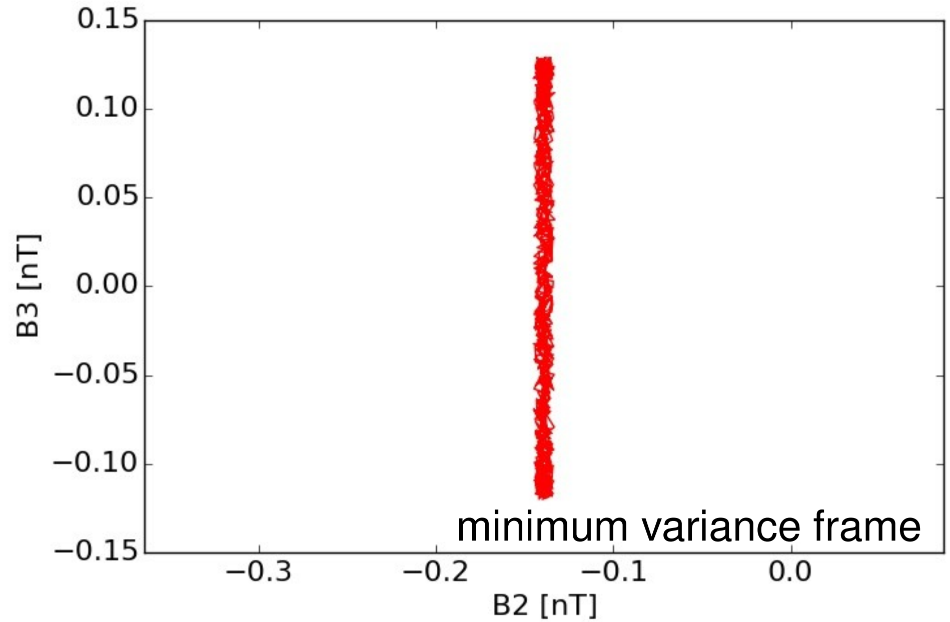
Generate time series, add noise then transform it to a randomly oriented frame

$$B_x = B_0$$

$$B_y = 0$$

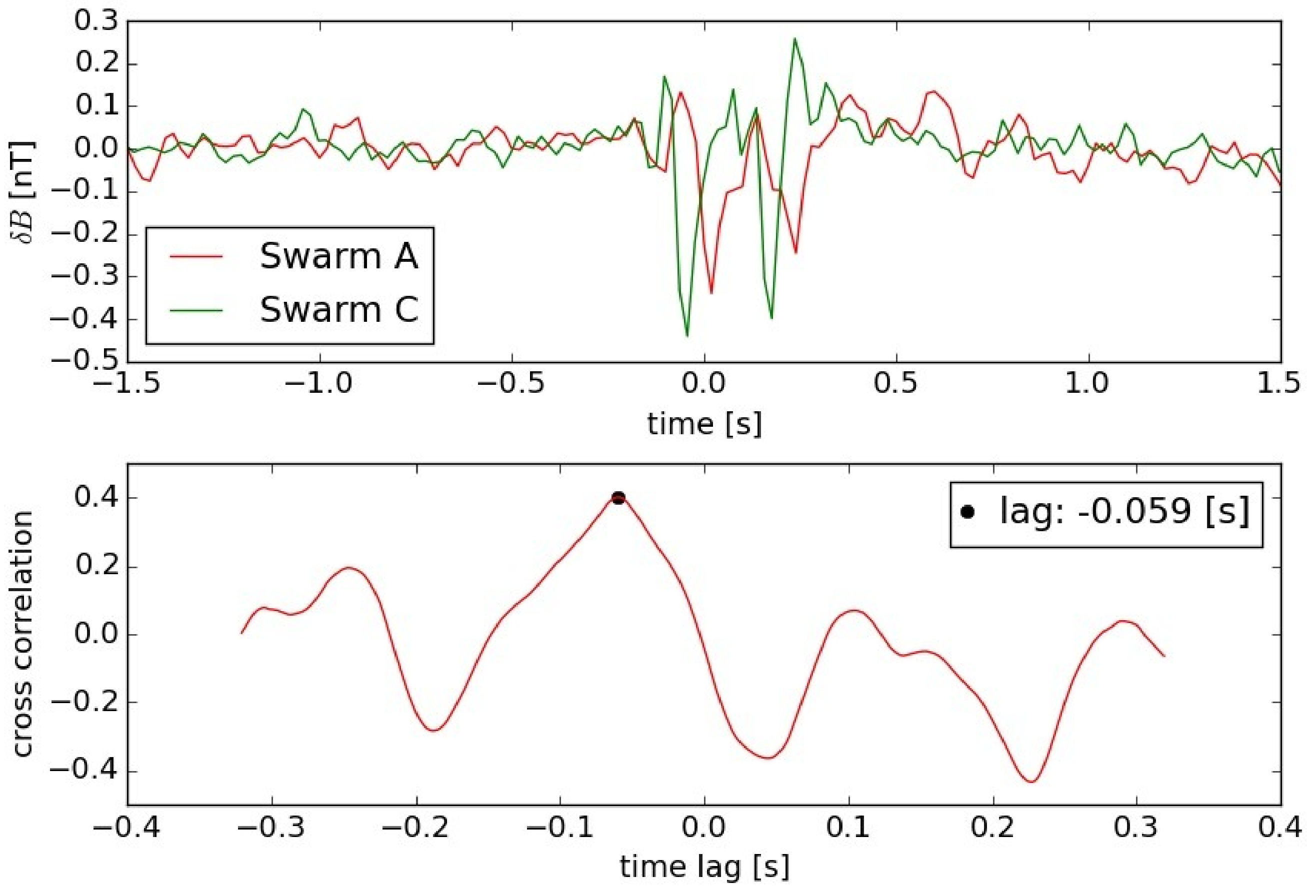
$$B_z = \delta B \cos(2\pi t)$$





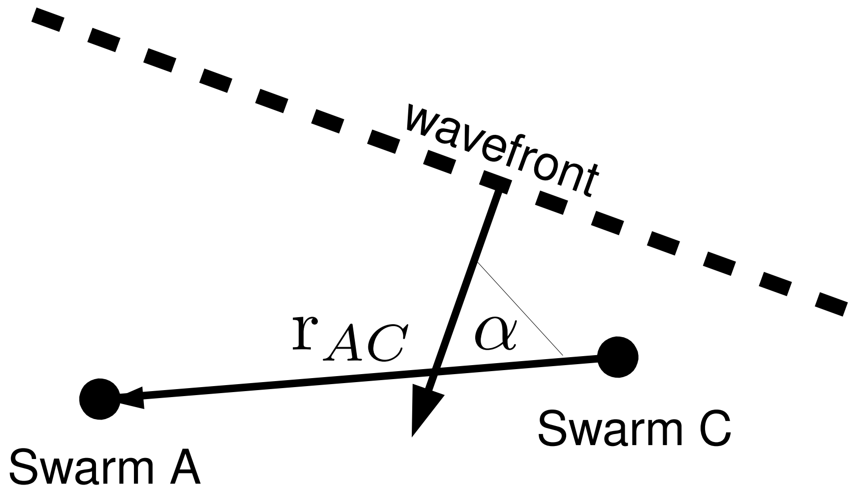
Cross-correlation function

$$K_{XY}(\tau) = \mathrm{E}[(X_t - \mu_X)(Y_{t+\tau} - \mu_Y)]$$



Time lag corresponding to maximum cross-correlation gives average shift (lag) between time series

Inter-satellite cross-correlation and propagation speed



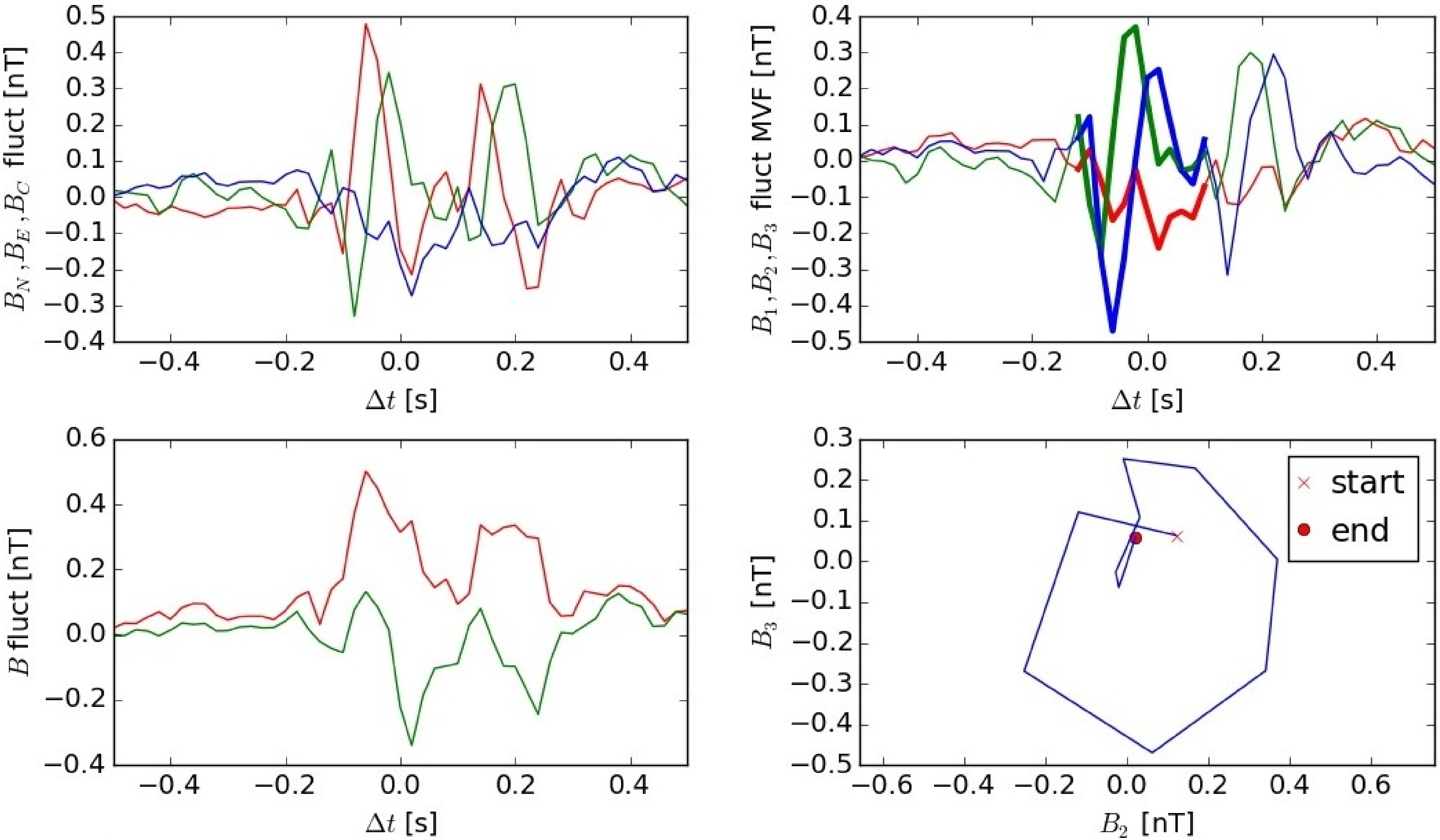
$$V_W = \left| \cos\alpha \frac{|\mathbf{r}_{AC}|}{\Delta t} \right|$$

In current calculations all uncertainties sit here, in the minimum variance contribution

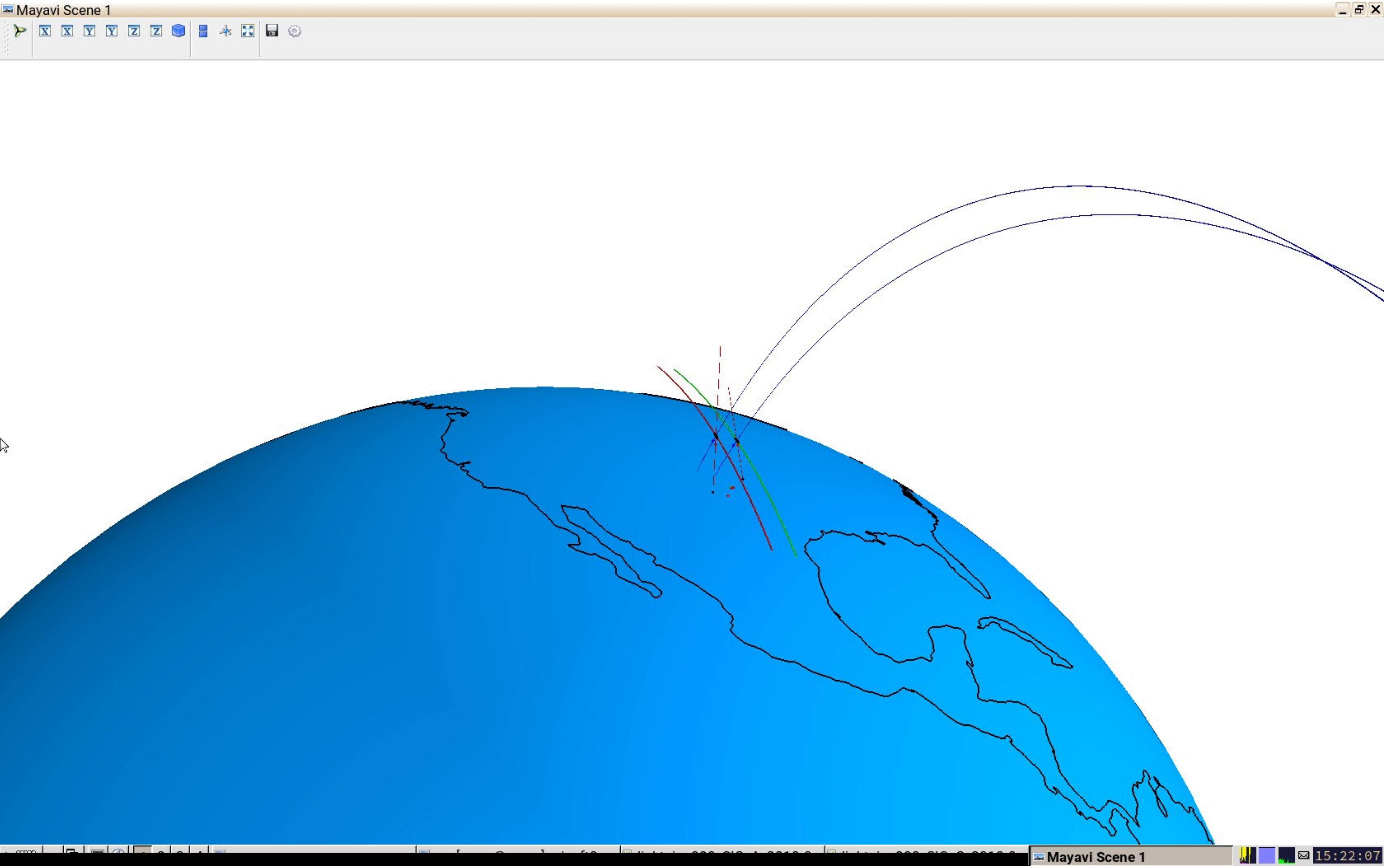
Having
- the wavevector from minimum variance method
- time lag from inter-satellite cross-correlation analysis
- inter-satellite distance from the Swarm positions

we can estimate the propagation speed

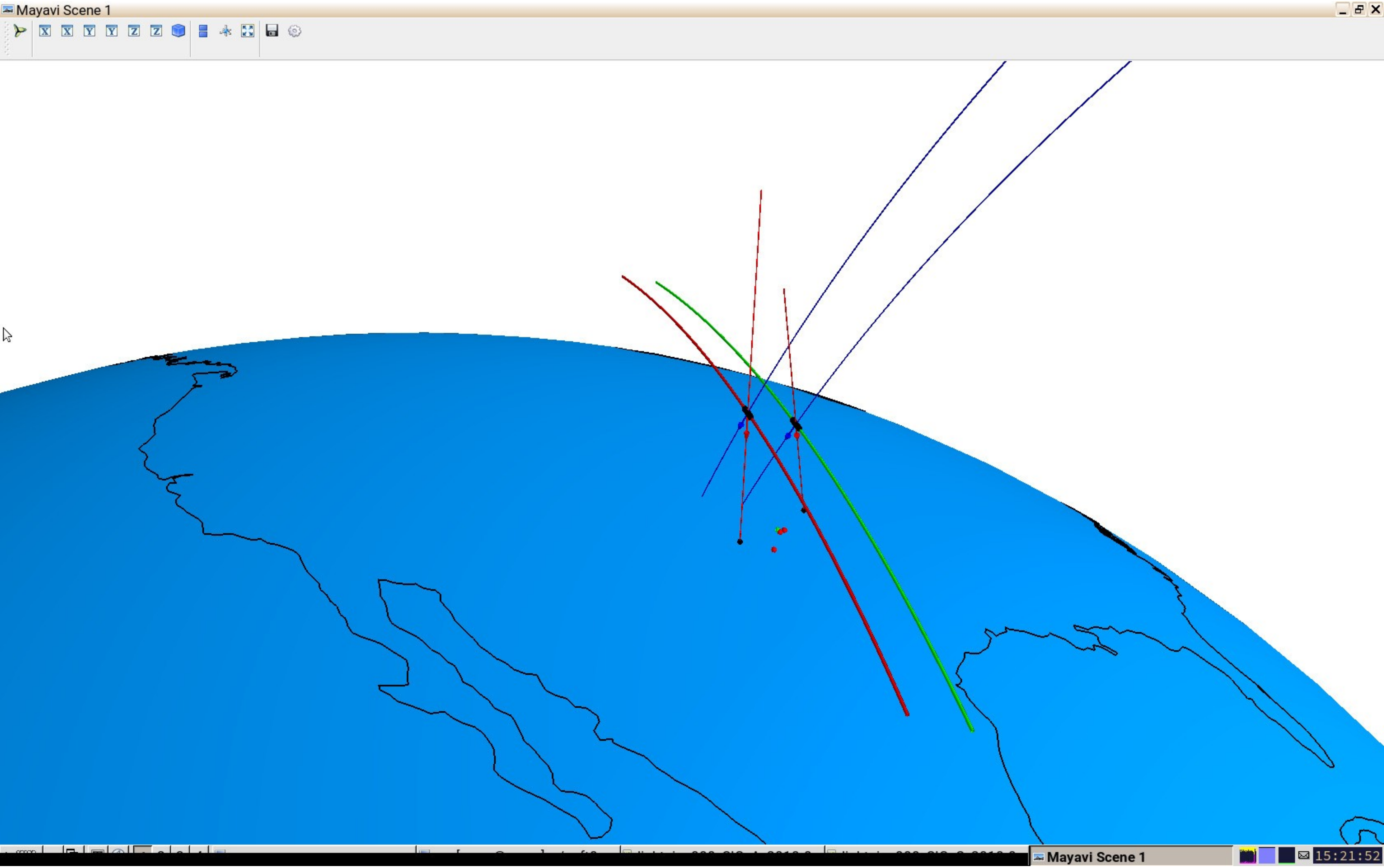
USA, 2018-05-20T06:05:20.888000, minimum variance analysis



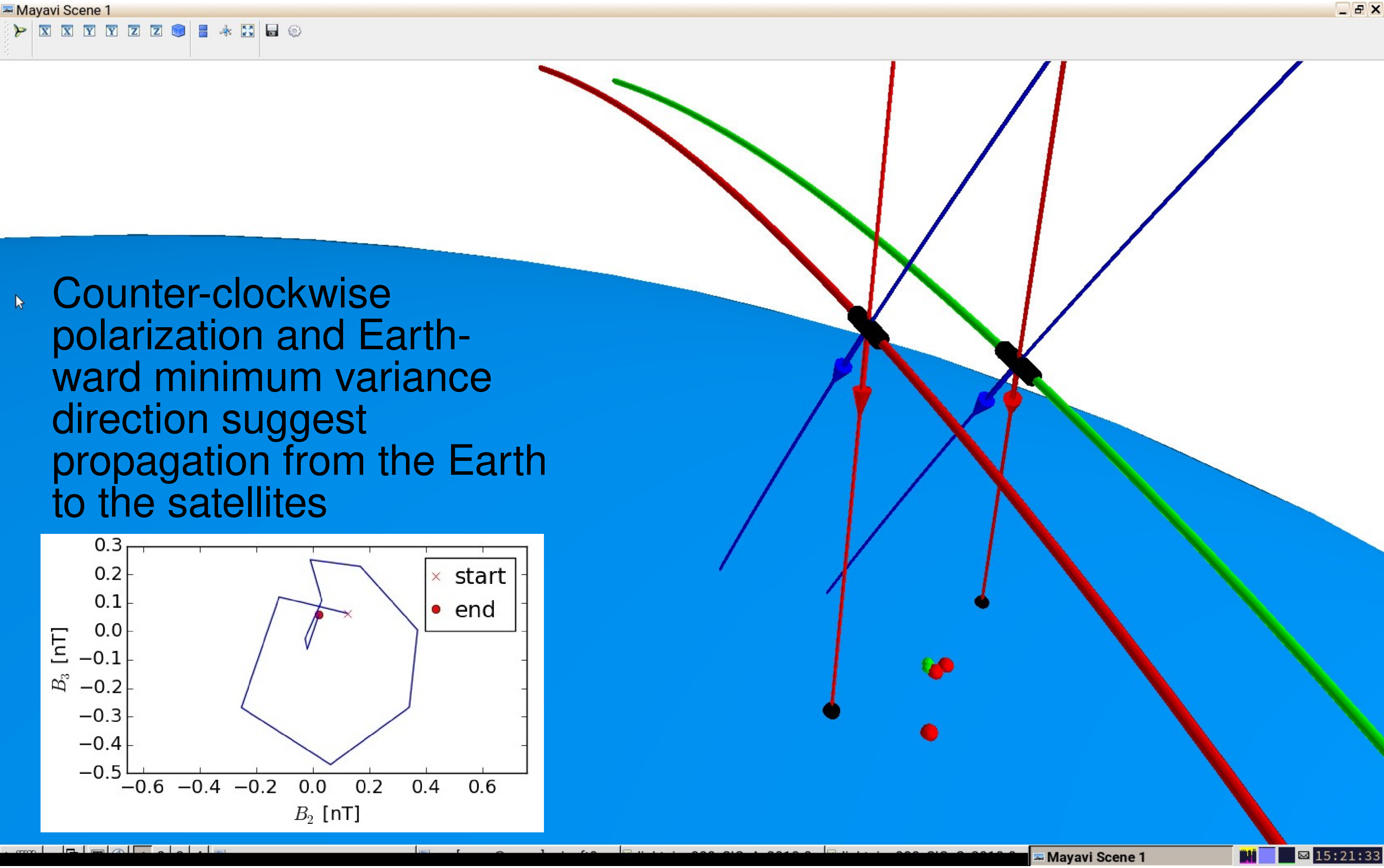
USA, 2018-05-20T06:05:20.888000, distant view



USA, 2018-05-20T06:05:20.888000, closer view

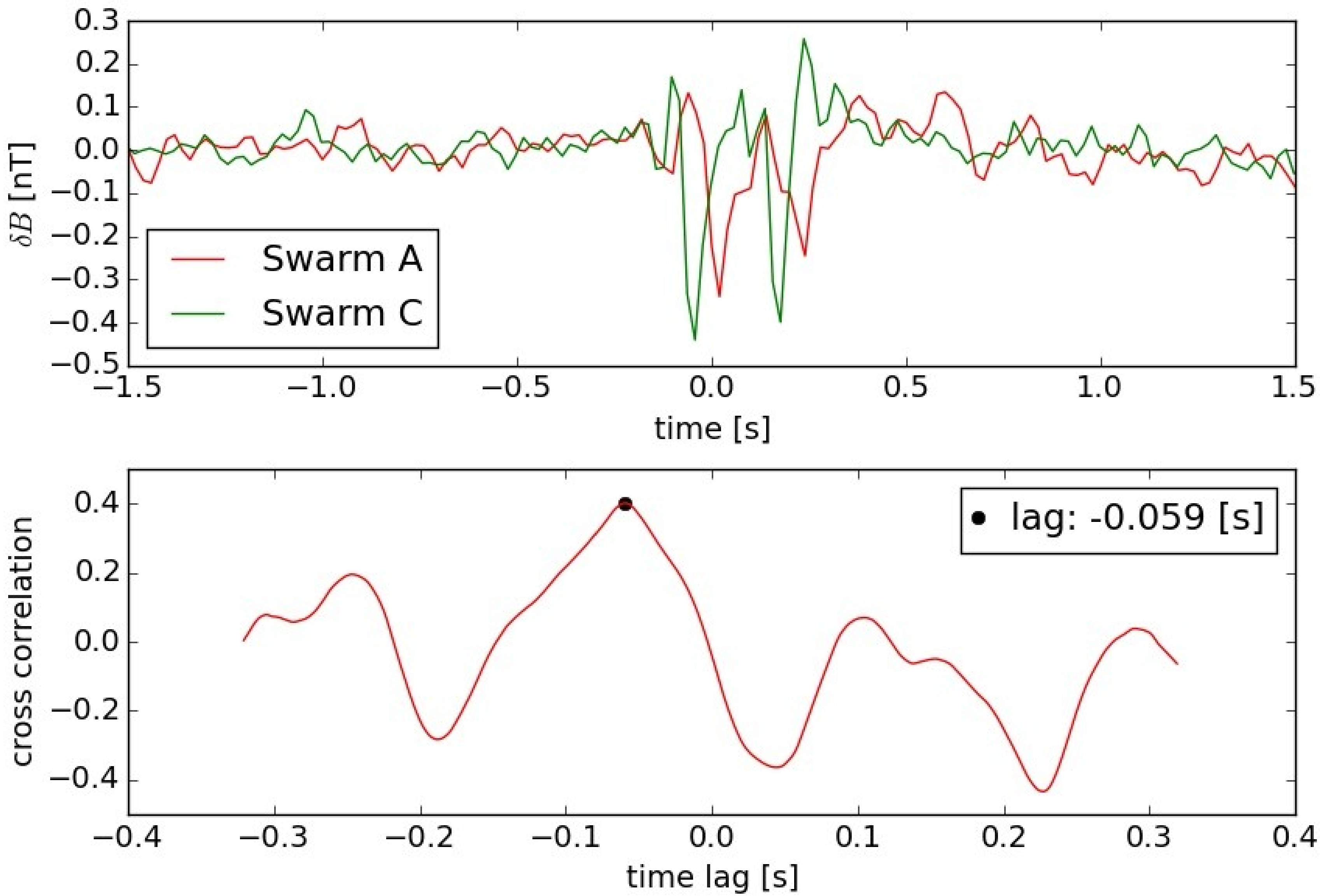


USA, 2018-05-20T06:05:20.888000, close-up



Counter-clockwise polarization and Earth-ward minimum variance direction suggest propagation from the Earth to the satellites

USA, 2018-05-20T06:05:20.888000, cross-correlation analysis



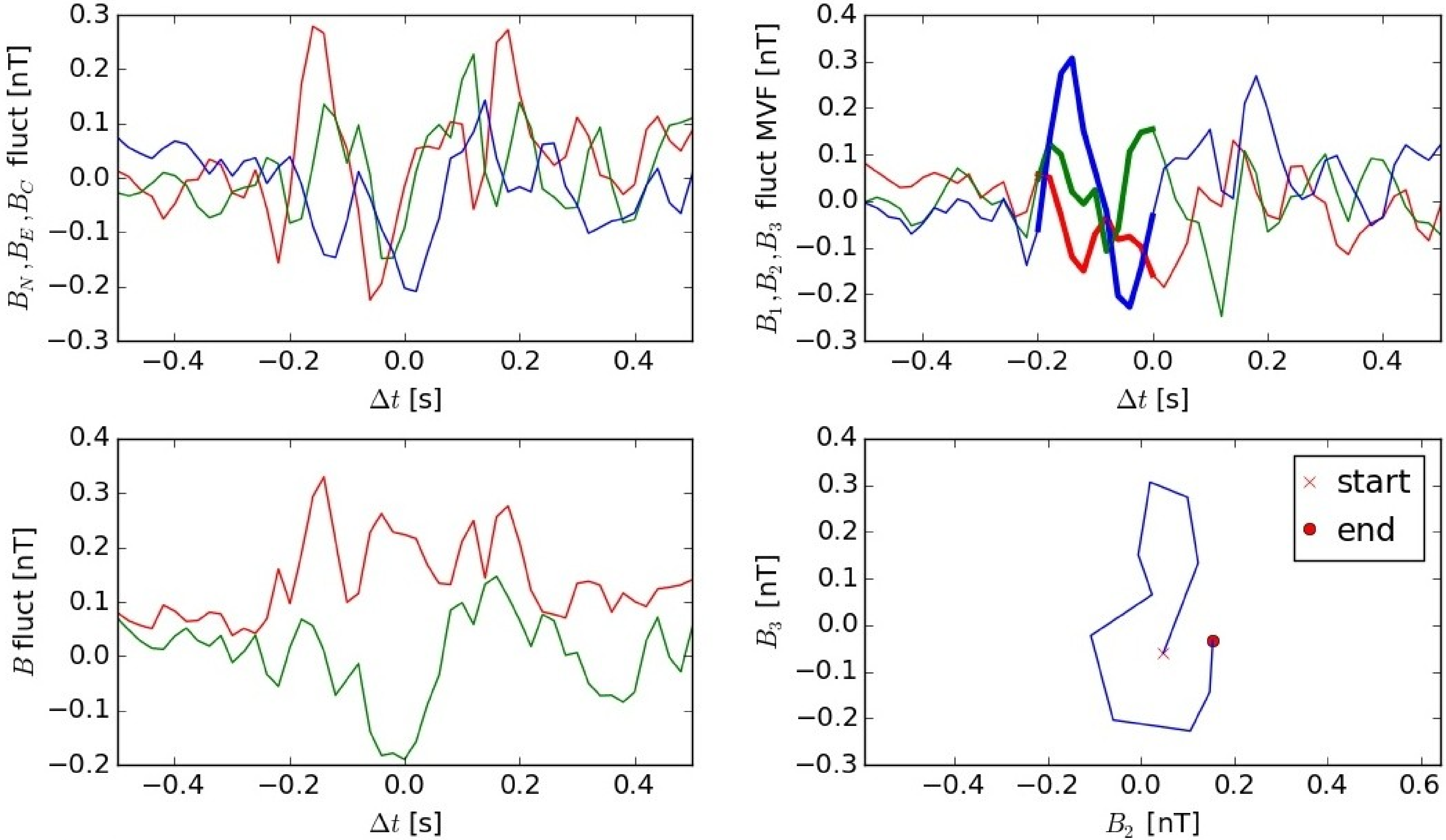
USA, 2018-05-20T06:05:20.888000, summary listing

Swarm A, 2018-05-20 06:05:20.888000, lon -103.38 deg, lat 33.79 deg, alt 441.78 km
minvar_dir NEC [-0.10,0.16,0.98] +/- [0.20,0.21,0.11]
eigenvalues [1.00,5.27,7.96]
(B,minvar_dir) angle 34.38 +/- 11.44 deg
lightnings [0.038, 0.182, 0.542] s

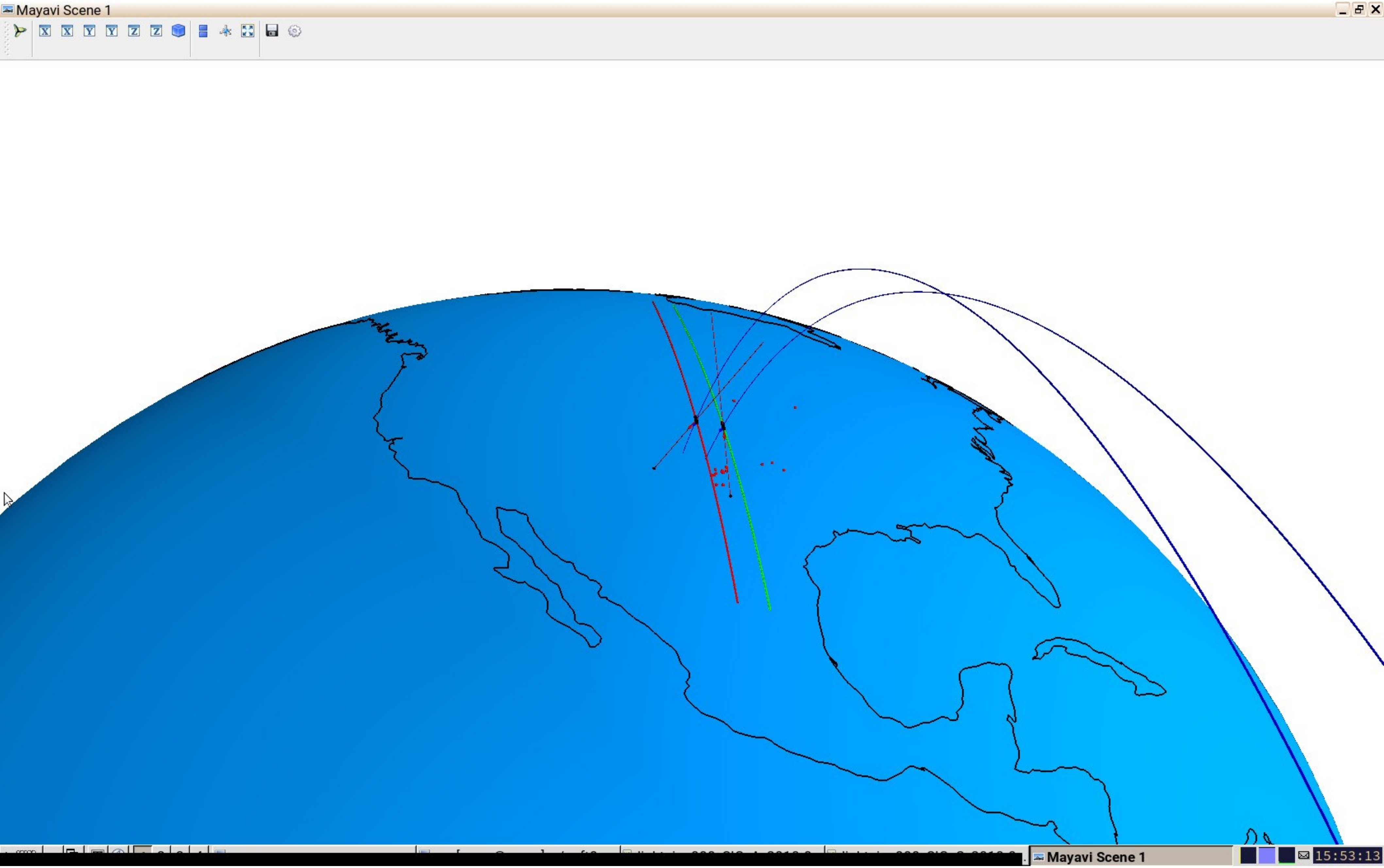
Swarm C, 2018-05-20 06:05:21.065000, lon -101.93 deg, lat 33.35 deg, alt 441.75 km
minvar_dir NEC [0.40,0.46,0.79] +/- [0.28,0.15,0.18]
eigenvalues [1.00,5.10,10.88]
(B,minvar_dir) angle 24.67 +/- 13.36 deg
lightnings [0.041, 0.053, 0.215, 0.359, 0.719] s

Swarm A-C distance 153.54 km, lag -0.0593 s
cross-correlation speed 677.17 +/- 358.64 km/s

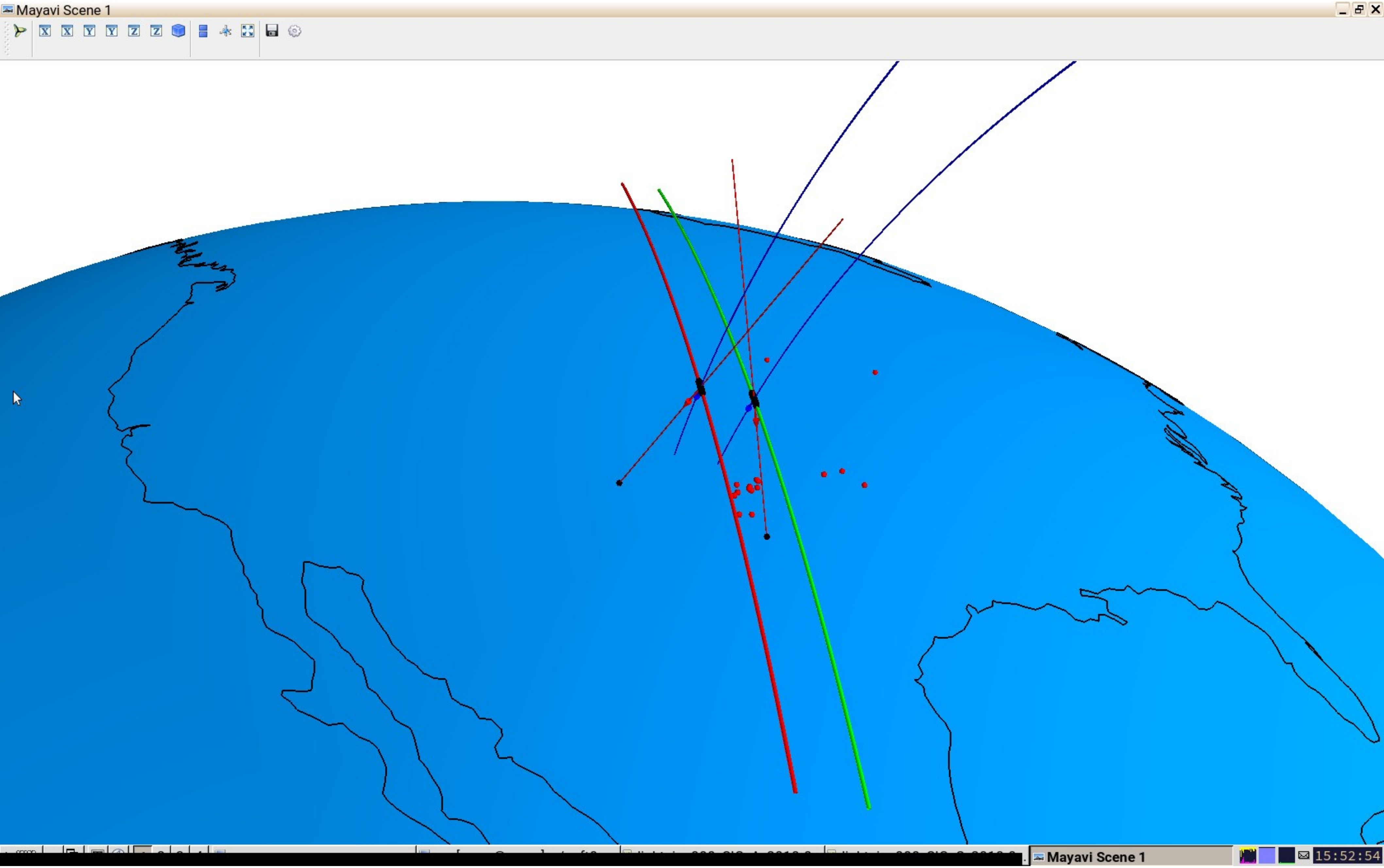
USA, 2018-05-20T06:05:26.227000, minimum variance analysis



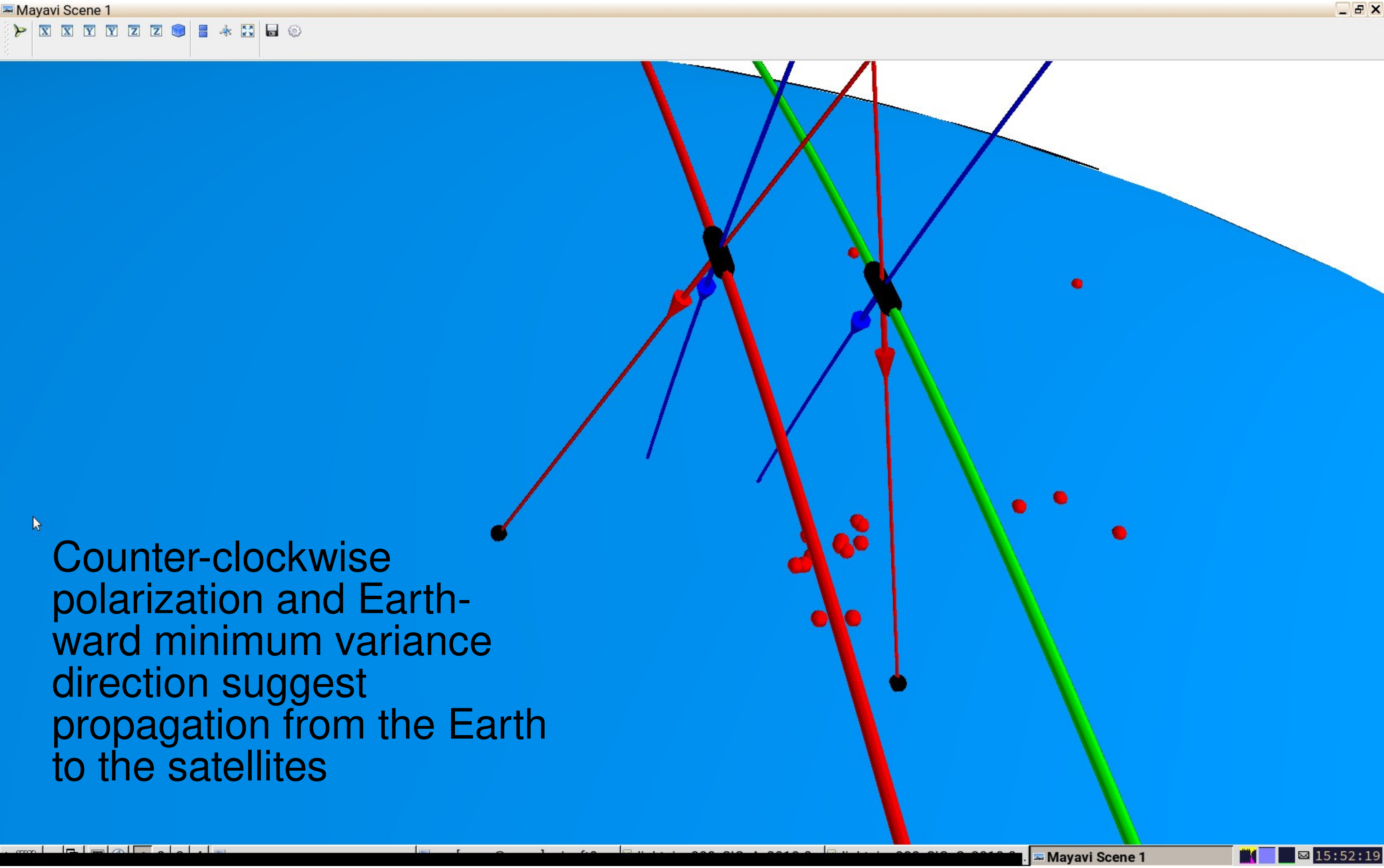
USA, 2018-05-20T06:05:26.227000, distant view



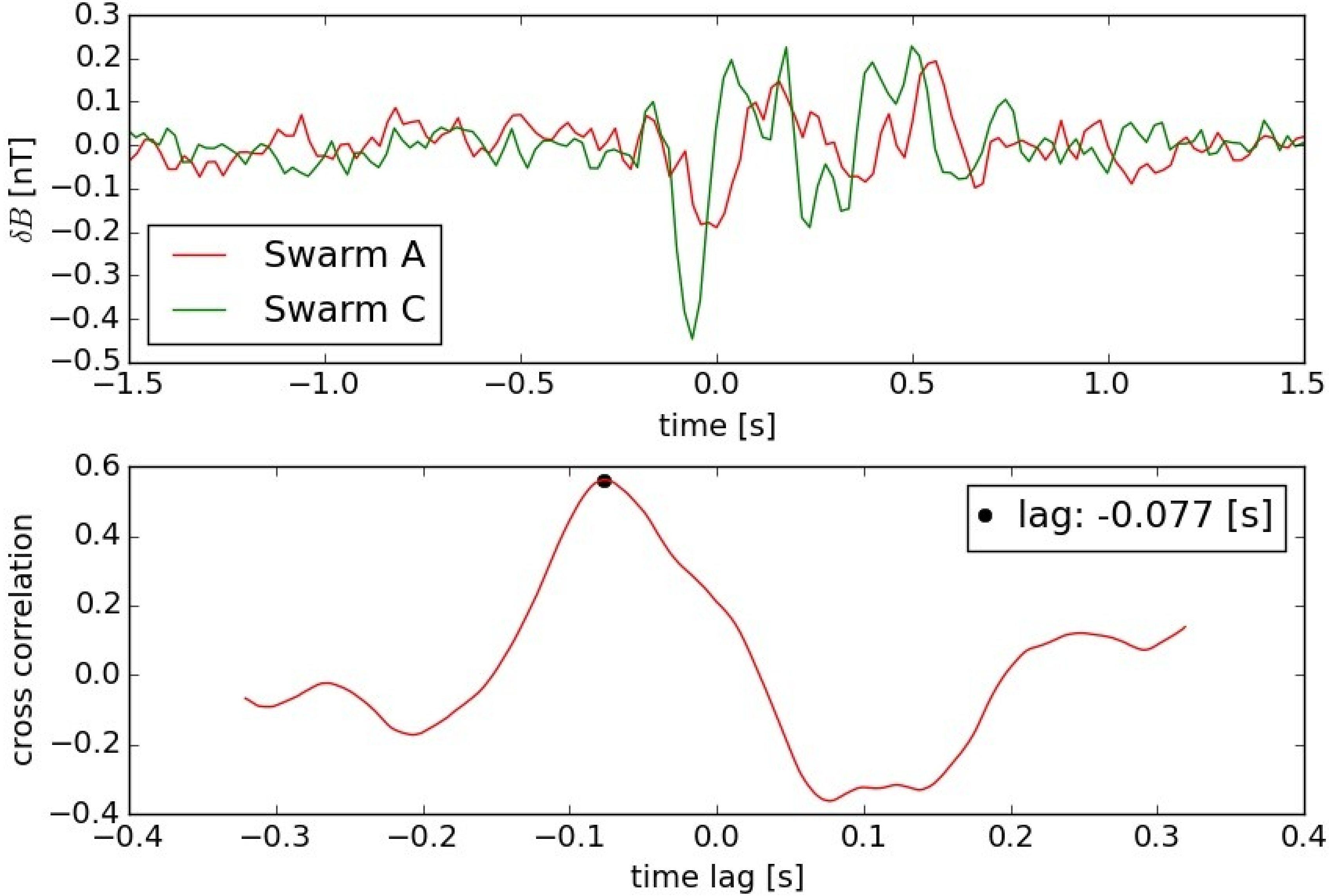
USA, 2018-05-20T06:05:26.227000, closer view



USA, 2018-05-20T06:05:26.227000, close-up



USA, 2018-05-20T06:05:26.227000, cross-correlation analysis



USA, 2018-05-20T06:05:26.227000, summary listing

Swarm A, 2018-05-20 06:05:26.227000, lon -103.38 deg, lat 33.45 deg, alt 441.76 km
minvar_dir NEC [0.21,-0.34,0.92] +/- [0.24,0.51,0.26]
eigenvalues [1.00,1.43,6.44]
(B,minvar_dir) angle 27.52 +/- 17.13 deg
lightnings [0.117, 1.217, 1.281, 1.849, 2.321, 3.301, 3.325, 4.145, 4.225, 4.575, 4.577, 5.081, 5.085, 5.203, 5.215, 5.377, 5.521, 5.881] s
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Swarm C, 2018-05-20 06:05:26.165000, lon -101.93 deg, lat 33.02 deg, alt 441.74 km
minvar_dir NEC [-0.20,0.23,0.95] +/- [0.17,0.37,0.19]
eigenvalues [1.00,2.34,12.99]
(B,minvar_dir) angle 41.46 +/- 11.72 deg
lightnings [0.055, 1.155, 1.219, 1.787, 2.259, 3.239, 3.263, 4.083, 4.163, 4.513, 4.515, 5.019, 5.023, 5.141, 5.153, 5.315, 5.459, 5.819] s
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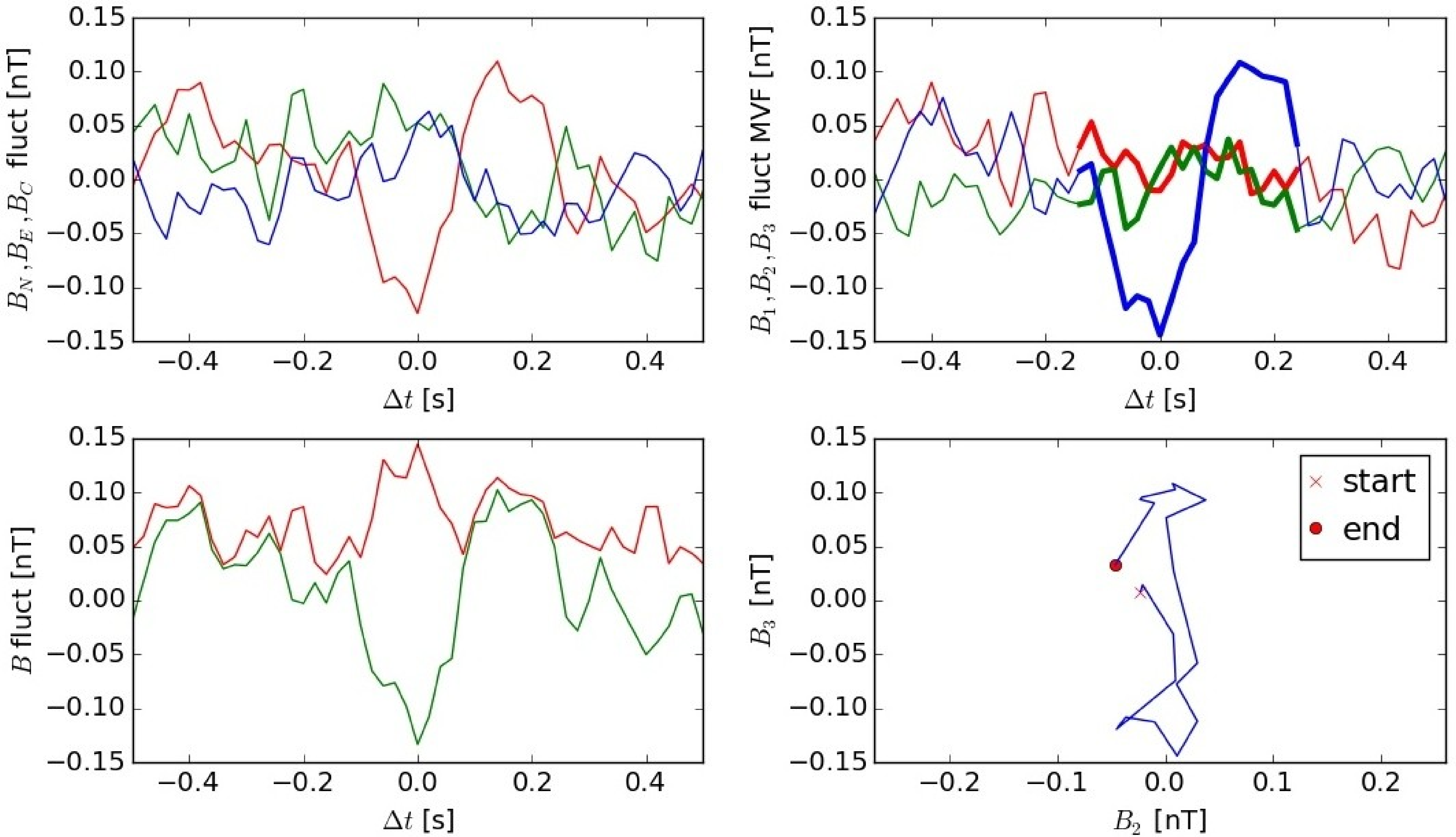
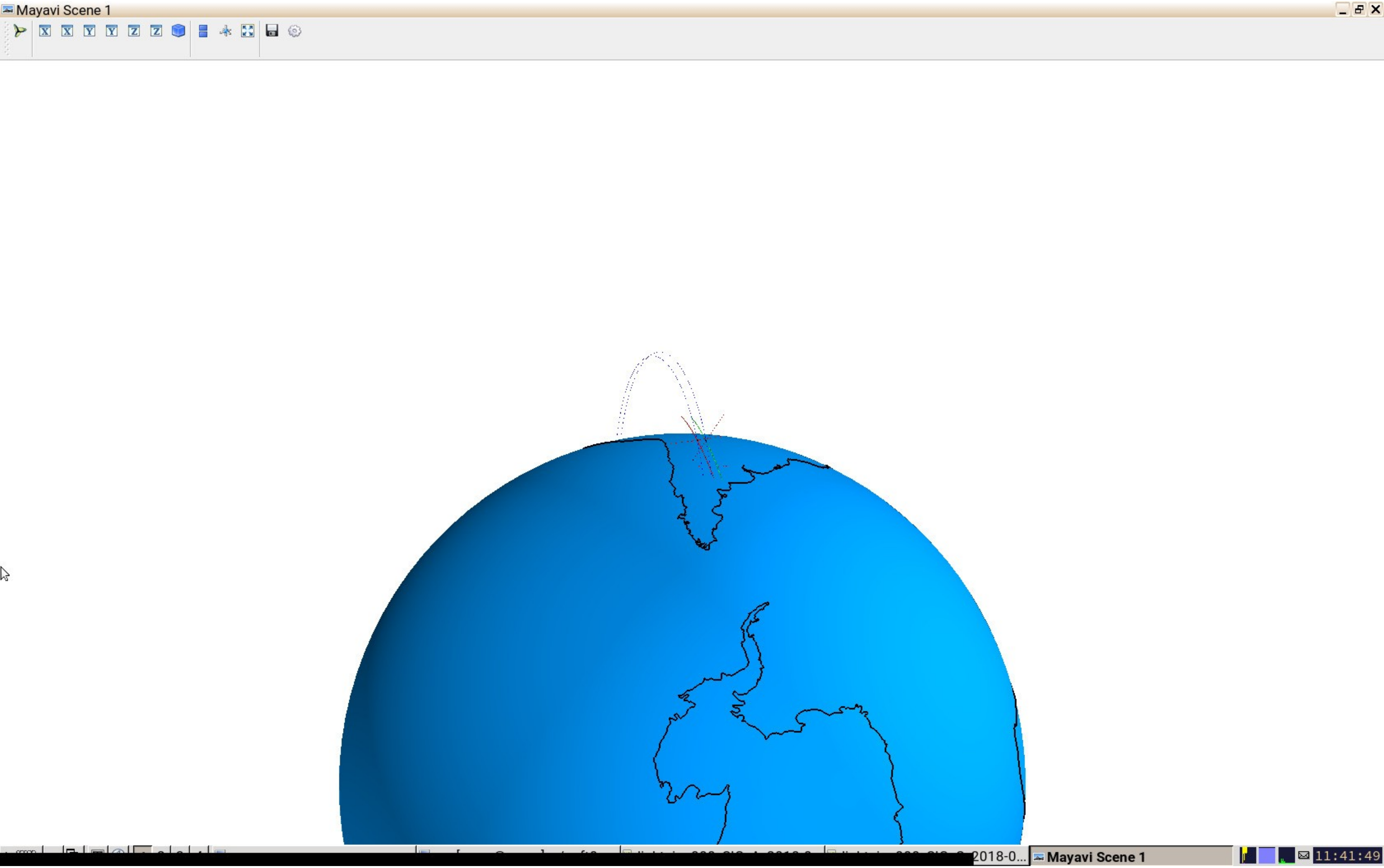
Swarm A-C distance 153.42 km, lag -0.0770 s
cross-correlation speed 584.54 +/- 373.01 km/s

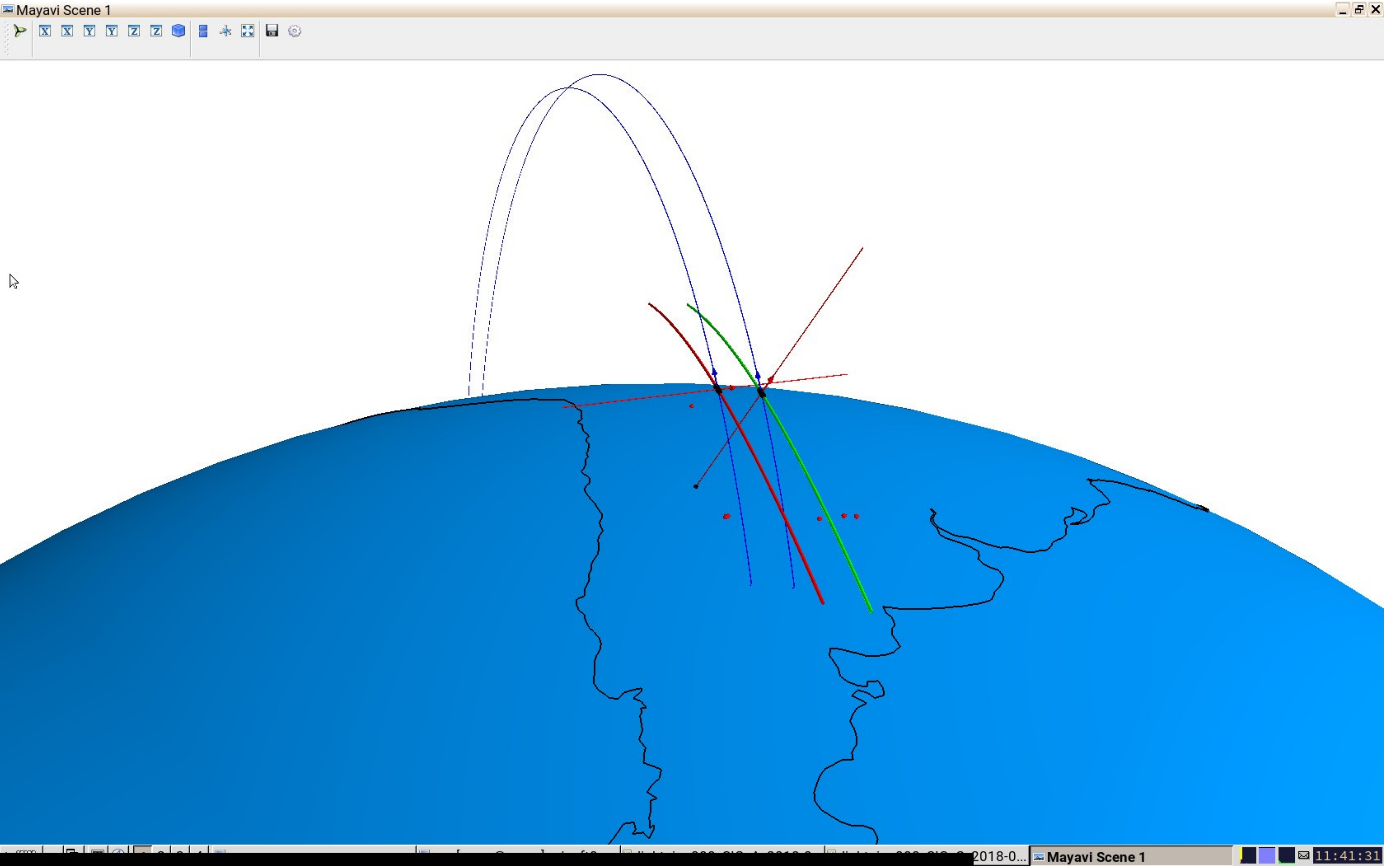
South America, 2019-03-04T01:59:06.046000
minimum variance analysis



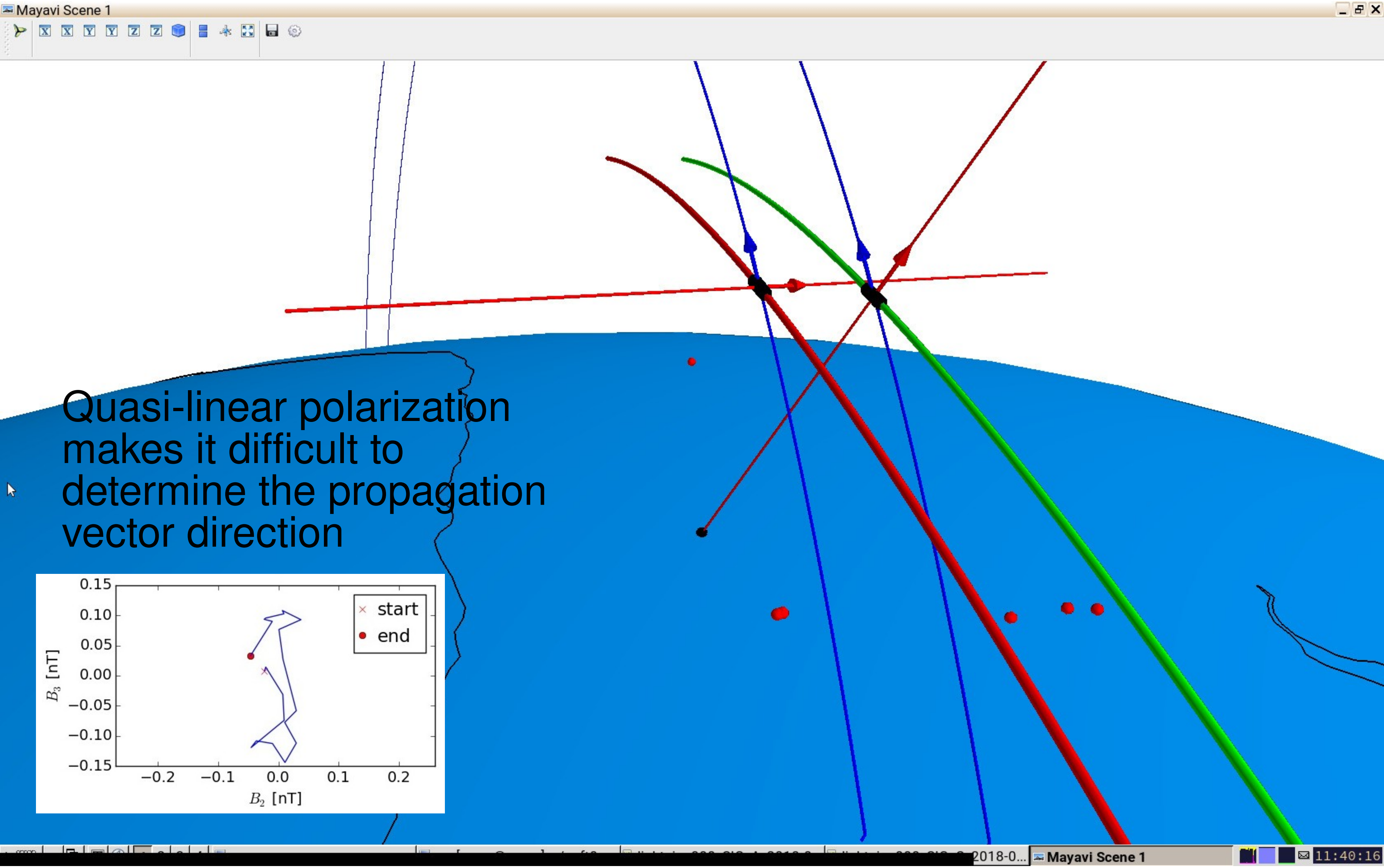
South America, 2019-03-04T01:59:06.046000, distant view



South America, 2019-03-04T01:59:06.046000, closer view

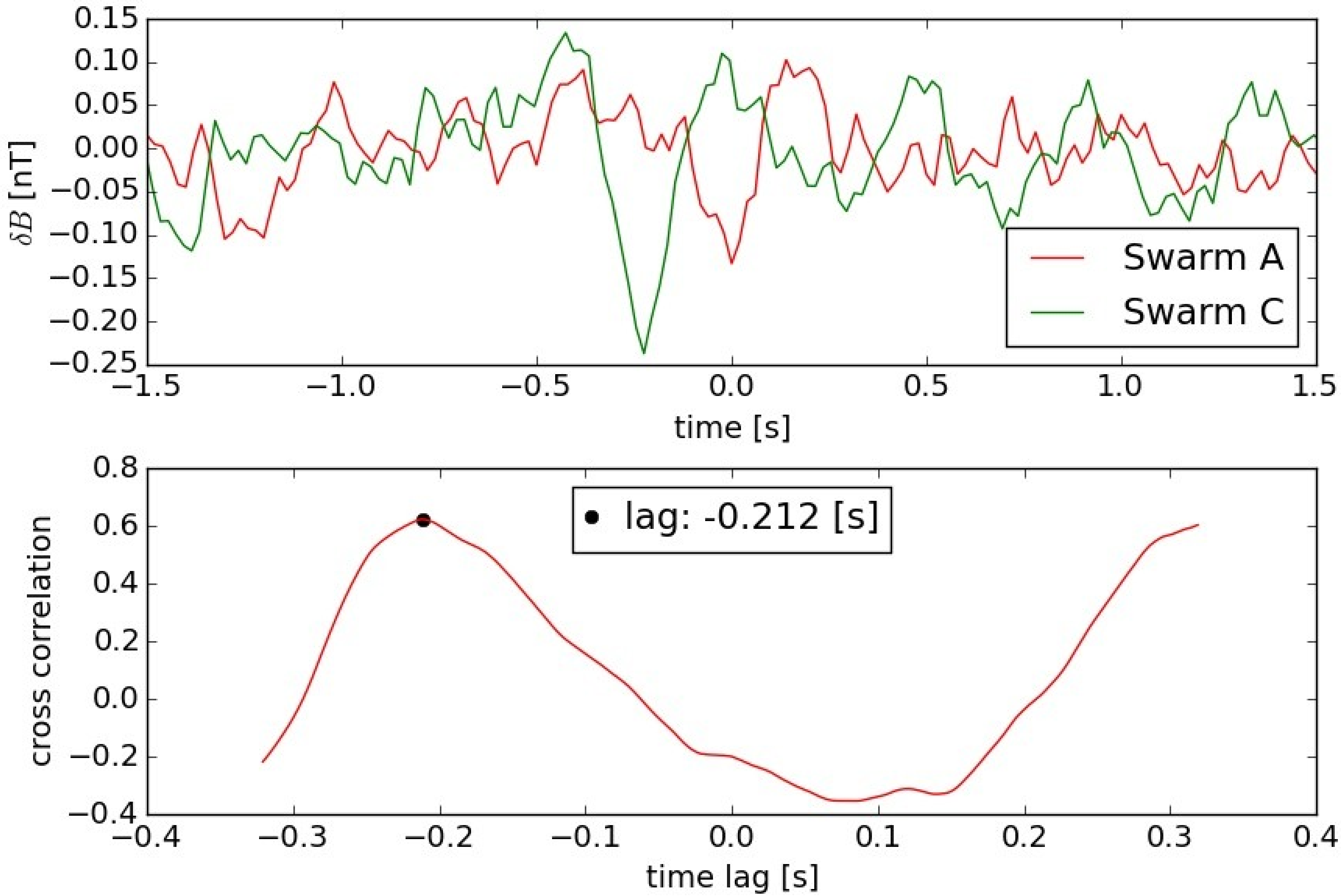


South America, 2019-03-04T01:59:06.046000, close-up



Quasi-linear polarization makes it difficult to determine the propagation vector direction

South America, 2019-03-04T01:59:06.046000,
cross-correlation analysis



South America, 2019-03-04T01:59:06.046000, summary listing

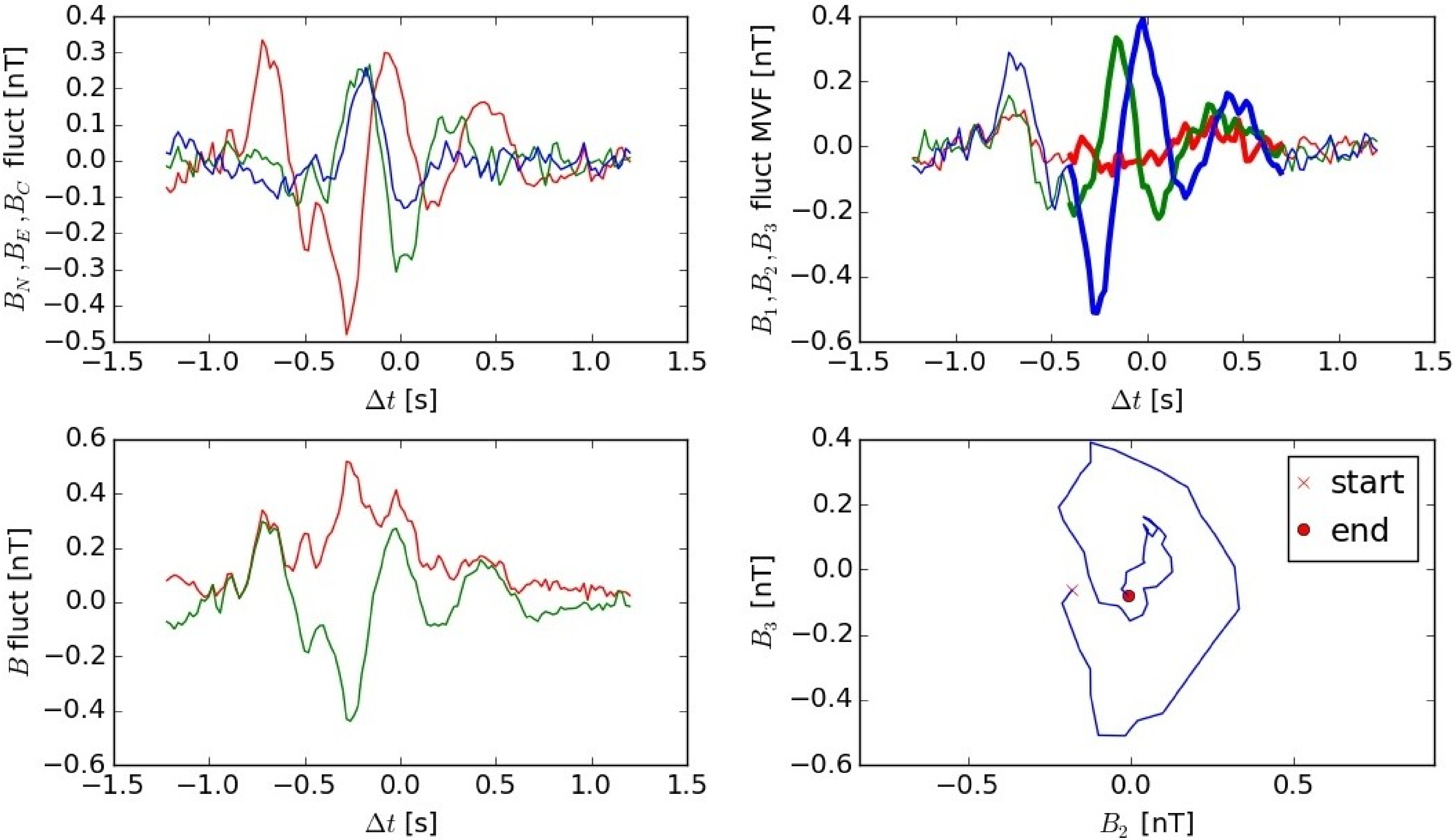
Swarm A, 2019-03-04 01:59:06.046000, lon -67.15 deg, lat -32.63 deg, alt 450.75 km
minvar_dir NEC [0.51,0.85,0.15] +/- [0.29,0.32,0.42]
eigenvalues [1.00,1.69,23.09]
(B,minvar_dir) angle 72.41 +/- 7.74 deg
lightnings [0.672, 1.603, 3.672, 4.179, 4.925, 5.676] s

Swarm C, 2019-03-04 01:59:05.821000, lon -65.69 deg, lat -33.02 deg, alt 450.88 km
minvar_dir NEC [-0.39,0.31,-0.87] +/- [0.19,0.47,0.28]
eigenvalues [1.00,1.63,29.42]
(B,minvar_dir) angle 79.07 +/- 3.37 deg
lightnings [0.447, 1.378, 3.447, 3.954, 4.7, 5.451] s

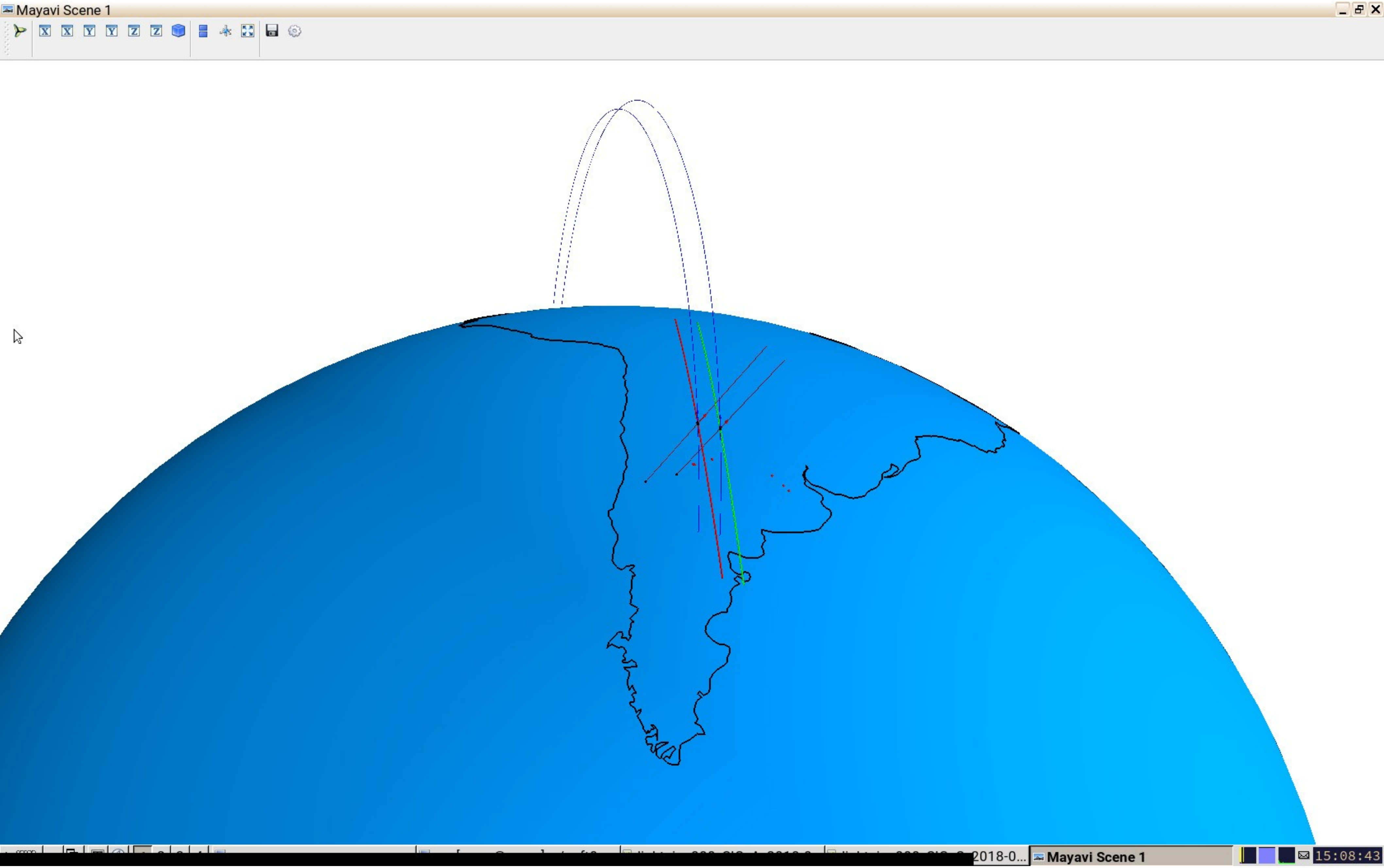
Swarm A-C distance 153.01 km, lag -0.2116 s
cross-correlation speed 293.61 +/- 127.22 km/s

Slower wave as compared with previous ones

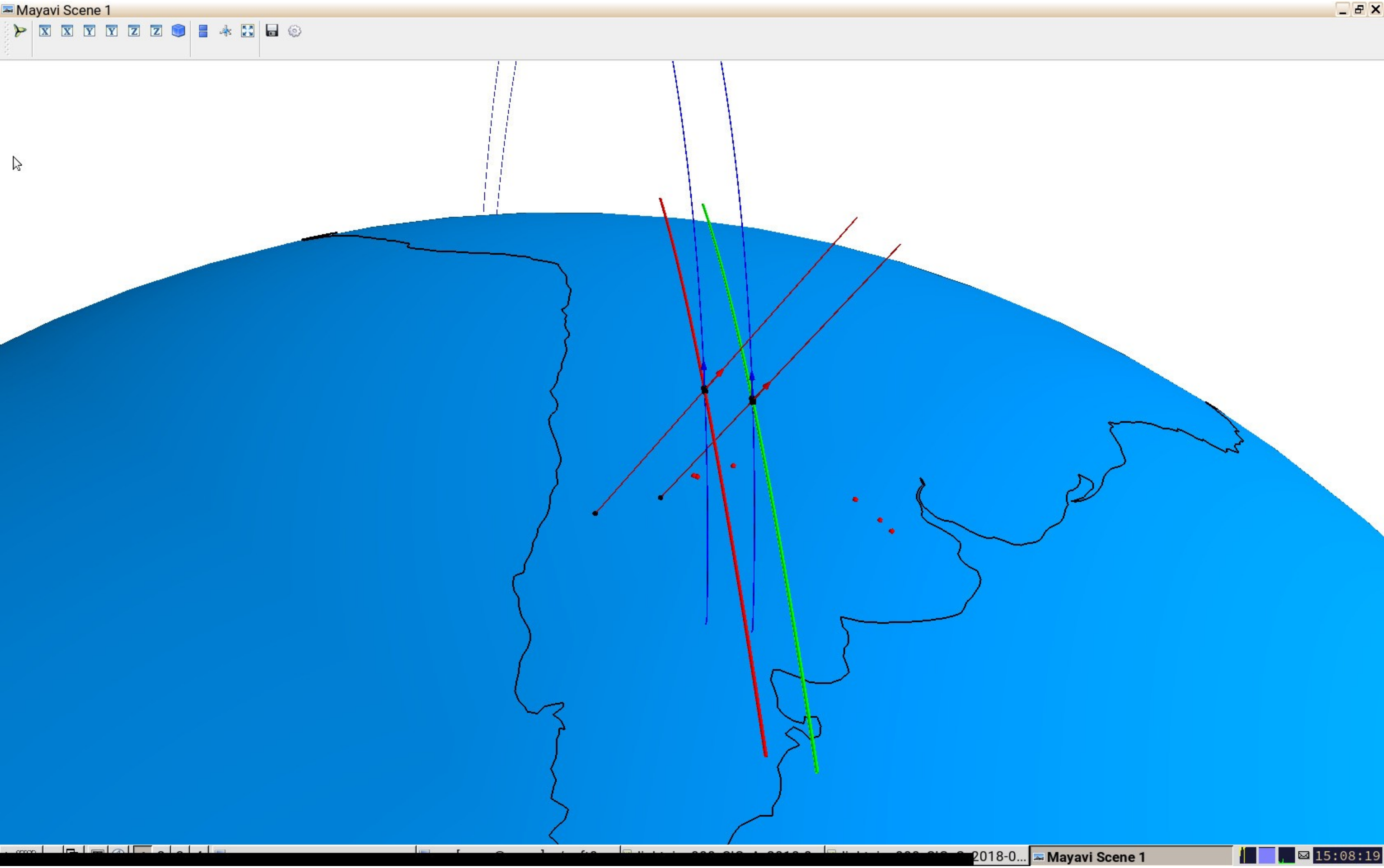
South America, 2019-03-04T01:59:36.245000,
minimum variance analysis



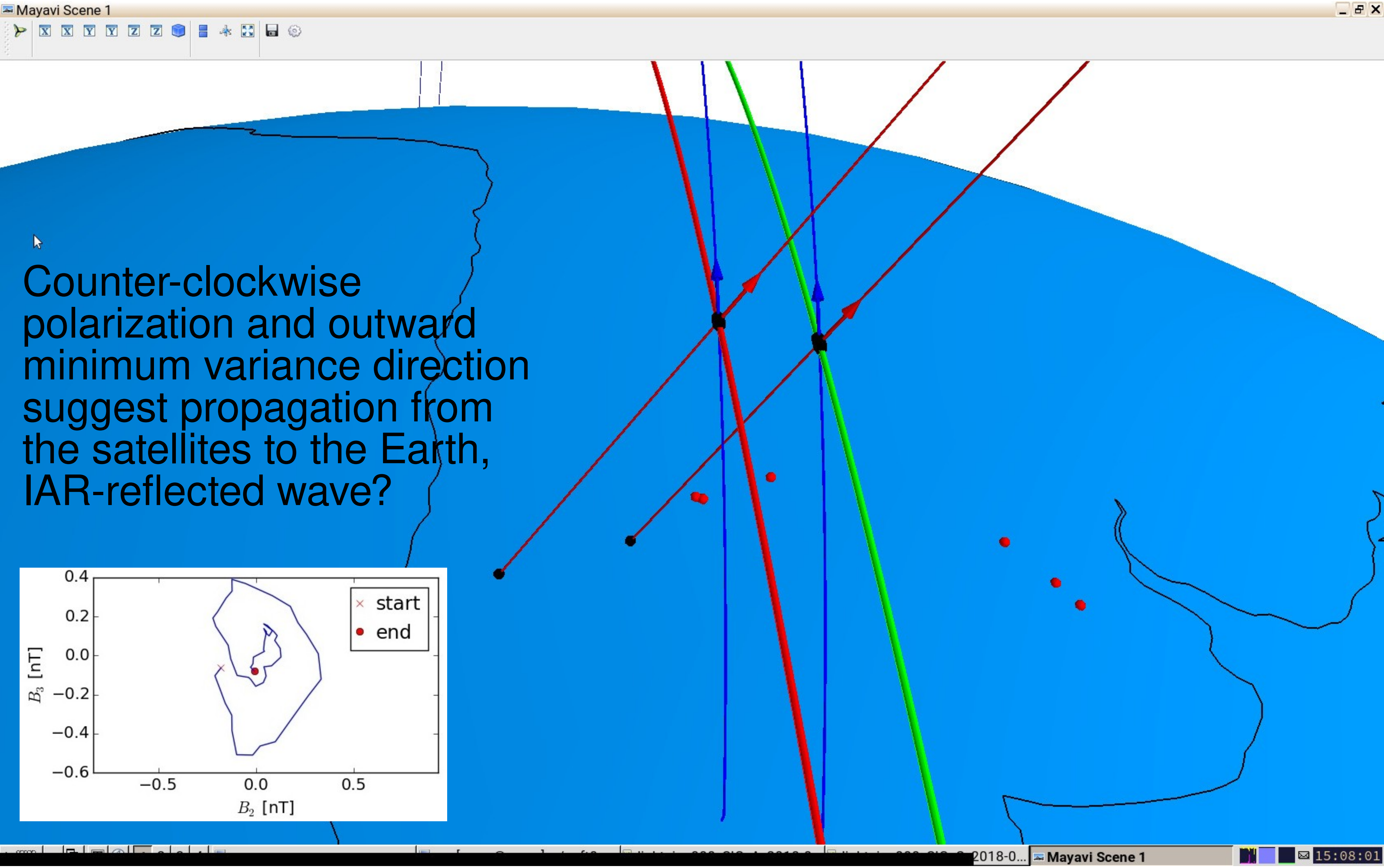
South America, 2019-03-04T01:59:36.245000, distant view



South America, 2019-03-04T01:59:36.245000, closer view

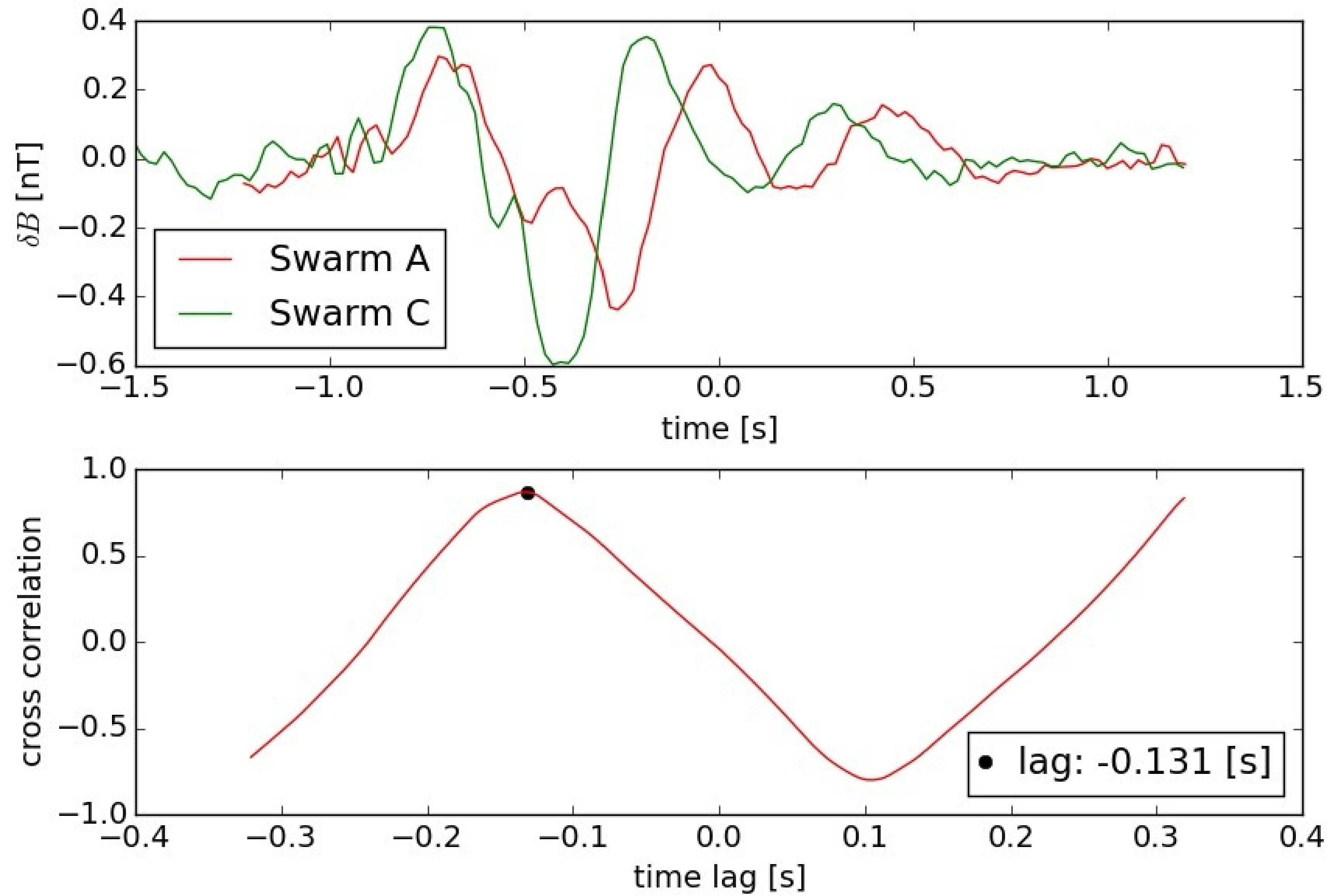


South America, 2019-03-04T01:59:36.245000, close-up



Counter-clockwise polarization and outward minimum variance direction suggest propagation from the satellites to the Earth, IAR-reflected wave?

South America, 2019-03-04T01:59:36.245000,
cross-correlation analysis



South America, 2019-03-04T01:59:36.245000, summary listing

Swarm A, 2019-03-04 01:59:36.245000, lon -67.14 deg, lat -34.57 deg, alt 451.39 km
minvar_dir NEC [0.11,0.55,-0.83] +/- [0.03,0.04,0.03]
eigenvalues [1.00,10.75,27.07]
(B,minvar_dir) angle 54.31 +/- 1.73 deg
lightnings [6.666, 10.819, 11.635, 12.62, 12.762, 13.242] s

Swarm C, 2019-03-04 01:59:35.879000, lon -65.68 deg, lat -34.94 deg, alt 451.51 km
minvar_dir NEC [-0.10,0.44,-0.89] +/- [0.04,0.07,0.03]
eigenvalues [1.00,5.19,19.91]
(B,minvar_dir) angle 62.13 +/- 1.68 deg
lightnings [6.3, 10.453, 11.269, 12.254, 12.396, 12.876] s

Swarm A-C distance 149.69 km, lag -0.1315 s
cross-correlation speed 547.49 +/- 40.59 km/s

Large time lag between identified nearby lightnings and the Swarm detection is consistent with propagation toward the Earth, possible explanation: reflection from IAR

Results of the minimum variance and cross-correlation analysis: summary

The ratio of the maximum and minimum eigenvalues indicates that plane-wave model generally applies to the analyzed cases

In many analyzed cases the inferred wave-propagation direction points approximately towards lightnings suggesting a possible connection; realistic raytracing would be useful here

The angle between the inferred wave propagation and the magnetic vector is

- 25-65 deg for circularly polarized waves,
- 70-80 deg for linearly polarized wave (TBC if generic)

which suggests oblique propagation wrt magnetic field

Generally the waves are found to propagate outward from the Earth, in one case a wave is found to propagate toward the Earth (possible IAR reflection)

Wave velocities from inter-satellite cross-correlation analysis:
~500-700 km/s for circular polarization
~300 km/s for linearly polarized waves (TBC if generic)